A nonlinear Dirac equation of Yamabe type and constant mean curvature surfaces

Bernd Ammann
MSRI
17 Gauss Way
Berkeley, CA 94720-5070
U.S.A.

Abstract

The subject of the talk is a conformally invariant functional on a compact Riemannian spin manifold (M^n, g, σ). We show that the functional attains its infimum if the first positive eigenvalue $\lambda_1^+(M, g, \sigma)$ of the Dirac operator on (M, g, σ) satisfies

$$\lambda_1^+(M, g, \sigma)\mathrm{vol}(M, g)^{1/n} < \lambda_1^+(S^n)\mathrm{vol}(S^n)^{1/n}.$$

We provide two applications of this result. As a first application, using the spinorial Weierstrass representation, the minimizer provides periodic constant mean curvature surfaces in the three-dimensional spaces of constant curvature R^3, S^3 and H^3. As a second application, we show that the first Dirac eigenvalue is bounded from below in a spin-conformal class, and the infimum is attained by a metric with certain singularities.

In the last part of the talk, we present some theorems showing that the above inequality holds on a large class of manifolds, including all compact Riemann surfaces of genus ≥ 1 with a suitable spin structure, all non-conformally-flat spin manifolds of dimension ≥ 7, and all conformally flat spin manifolds whose Dirac operator has a non-trivial mass endomorphism or a non-trivial kernel.

The subject of the talk has many analogies and relations to the solution of Yamabe's problem, which amounts to finding a constant scalar curvature metric in a given conformal class. In particular, the

mass endomorphism mentioned above is a spinorial version of the ADM mass that appears in the solution of Yamabe's problem.