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An infinitary approach to (hyper)graph regularity and removal

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Abstract

The famous Szemeredi regularity lemma gives a structural theorem for very large (but finite) dense graphs, which then has many applications to such graphs, for instance in being able to efficiently eradicate all copies of a given subgraph by edge removal if the original number of such copies was small. These results have been extended to hypergraphs (leading for instance to another proof of Szemeredi's theorem on arithmetic progressions) but the proofs, while elementary and finitary, are somewhat messy and lengthy in nature. Here we present an alternate "infinitary" route to these results, by passing from a sequence of large finite dense deterministic graphs to an infinite dense random graph, and analysing the resulting object instead. The advantage of doing this is that many of the "epsilon" quantities present in the finitary theory go to zero in the infinite limit, and one can now bring techniques from infinitary probability theory (in particular, the theory of conditional independence) to bear on the subject.