Additive Combinatorics
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Binary linear forms over finite sets of integers

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Abstract

Let A be a finite set of integers. For a polynomial $f(x_1, \ldots, x_n)$ with integer coefficients, let $f(A) = \{f(a_1, \ldots, a_n) \colon a_1, \ldots, a_n \in A\}$. It is proved that for most pairs of binary linear forms $f(x, y) = u_1 x + v_1 y$ and $g(x, y) = u_2 x + v_2 y$ with integral coefficients such that $(u_i, v_i) = 1$ and $u_i \geq |v_i| \geq 1$ for $i = 1, 2$, there exist finite sets of integers A and B such that $|f(A)| > |g(A)|$ and $|f(B)| < |g(B)|$.

This is joint work with Kevin O'Bryant, Brooke Orosz, Imre Ruzsa, Manuel Silva.