

Additive Combinatorics
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k -point configurations in sets of
positive density of \mathbb{Z}^n

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Abstract

We show that if $n \geq 4k + 1$ and $A \subseteq \mathbb{Z}^n$ is a set of upper density $\varepsilon > 0$, then — in a sense depending on ε — A contains all large “copies” of any k -dimensional simplex $\Delta = \{0, v_1, \dots, v_k\} \subset \mathbb{Z}^n$. A copy of Δ is a k -simplex Δ' obtained from Δ via translations, rotations and dilations. If time permits we discuss quantitative versions and related questions.

The proof relies on estimates for the corresponding theta function on the Siegel upper-half space, and on ideas from the proof of an analogous result on \mathbb{R}^n adapted to the discrete settings.