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Growth and generation in $SL_2(Z/pZ)$

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Abstract

We show that every subset of $\operatorname{SL}_2(Z/pZ)$ grows rapidly when it acts on itself by the group operation. It follows readily that, for every set of generators A of $\operatorname{SL}_2(Z/pZ)$, every element of $\operatorname{SL}_2(Z/pZ)$ can be expressed as a product of at most $O((\log p)^c)$ elements of the union of A and A^{-1} , where c and the implied constant are absolute.