

Additive Combinatorics
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Quadratic fourier analysis

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Abstract

I will give an introduction to “quadratic Fourier analysis”. This is a term used to describe a body of results which link the study of combinatorial objects defined by a pair of linear equations (say 4-term arithmetic progressions) to certain “quadratic” functions which can be described very explicitly.

I will develop the theory in a model case (the “finite field model”), which contains most of the ideas and is relatively light on technicalities. This represents a substantial use of some of the ideas of additive combinatorics which will have been laid out in the introductory lectures.

I will then give an application, proving a conjecture of Bergelson, Host and Kra. This is a kind of strong version of Szemerédi’s theorem for progressions of length 4.