

Additive Combinatorics
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A uniform set with few progressions of length 4

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Abstract

Szemerédi's theorem on arithmetic progressions is an example of a result that is very simple to state and surprisingly hard to prove. From one perspective, the reason it is so hard is that Fourier analysis, which works well for progressions of length 3, is inadequate for longer progressions. The reason for this is that there are sets that have Fourier transforms that resemble those of random sets, but which do not have the same number of progressions of length 4 as a random set. Until recently, however, the known examples all had too many progressions of length 4, so they did not truly demonstrate the inadequacy of Fourier analysis. In this talk I shall present an example of a highly Fourier-random set with very slightly too few progressions of length 4. Whether one can improve on the “very slightly” is still an open problem.