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# AN ADAPTIVE REMESHING STRATEGY FOR VISCOELASTIC FLUID FLOW PROBLEMS

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**Abstract**

The simulation of highly viscoelastic fluid flow problems is still plagued with the so-called high Weissenberg problem (HWP). The Weissenberg number $We$ is the ratio of the magnitude of the elastic forces to that of the viscous forces. Most (if not all) numerical methods lose convergence i.e. fail to generate any result and/or fail to generate convergent solutions with respect to mesh size, at small or moderate Weissenberg numbers, limiting their use in industrial flow situations. Recently, a log-conformation formulation was proposed by Fattal and Kupferman [1] and further implemented in Hulsen, Fattal and Kupferman [2]. The main idea is to formulate the constitutive equation in terms of the conformation tensor instead of the extra-stress tensor. More precisely, the constitutive equation is written in terms of the logarithm of the positive definite conformation tensor which is therefore well defined. This idea is new for viscoelastic fluid flow problems but was already used for the simulation of turbulent flow problems where the $\kappa - \varepsilon$ model is frequently rewritten in terms of the variables $\mathcal{K} = \log \kappa$ and $\mathcal{E} = \log \varepsilon$ as in Ilinca, Hétu and

Pelletier [3]. One of the advantages is that when recovering $\kappa = \exp(\mathcal{K})$ or $\varepsilon = \exp(\mathcal{E})$, the result is always positive which is not always the case with formulations using $\kappa$ and $\varepsilon$ directly. This formulation has also some good influence on the convergence properties of the numerical solver. Indeed, the logarithm of a variable presenting very steep variations is much easier to capture with a finite element discretisation. Adaptive methods are now used extensively in CFD codes. Starting from a numerical solution, the error is estimated and the mesh is modified so that the numerical solution respects some prescribed error level. In particular, the development of anisotropic meshes where elements can present very large aspect ratios has received a lot of attention in the last few years. Robust anisotropic adaptive strategies require reliable and consistently accurate solvers capable of dealing with strongly elongated elements appearing in unstructured meshes. This kind of solver was certainly not available for viscoelastic flows until recently. In this work, we will present an anisotropic adaptive remeshing method combined with a logconformation formulation for the solution of various viscoelastic fluid flow problems. We will show that this formulation is stable and allows for numerical solutions of viscoelastic fluid flow problems on unstructured meshes, which is hardly possible with classical formulations. Numerical results will be presented for the flow around a circular cylinder and convergence with mesh size will be analysed for the Oldroyd-B and the Giesekus models (see Guenette et al. [4]). Our adaptive method will then be applied to the numerical simulation of free surface problems such as those arising in fluid-structure interaction, flame propagation, foams, mixing, etc. The same log-conformation formulation is used and free surfaces are captured using a level-set method developed in Fortin and Benmoussa [5, 6]. In particular the deformation of viscoelastic fluid drops in shear flows will be analysed. A typical situation is that of a viscoelastic fluid drop immersed in a Newtonian fluid matrix which deforms under a shearing flow. The shape of the drop can become very complex and numerical methods must be able to cope with strong topology changes and steep parameter variations along these interfaces. Numerical results will be presented.

**Key Words:** *Adaptive method, FEM, Viscoelastic material, Free surface problems.*

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