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Multiple eigenvalues and symmetric hyperbolic systems

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Abstract

The solutions of symmetric hyperbolic systems can be expressed as superpositions of plane wave solutions. These are expressed in terms of the eigenvalues and eigenvectors of linear combinations of the coefficient matrices that occur in the equations. Multiple eigenvalues lead to non-smooth dependence of the eigenvectors and eigenvalues on the parameters; these create singularities of solutions that have physical significance. The following result is relevant:

Theorem Let A, B and C be three real, symmetric matrices of order n. If n is congruent 2 mod 4, there exist three real numbers a, b and c, not all zero, such that aA + bB + cC has multiple eigenvalues.

The first part of the talk deals with some topological notions, the second part raises some questions in algebraic geometry.