≪ Première rencontre mathématique Bavière–Québec ≫
30 novembre–3 décembre, 2009

"First Bavaria–Québec Mathematical Meeting"
November 30–December 3, 2009

Reconstruction and approximation of functions from samples

Gerhard Schmeisser

Mathematisches Institut
Universität Erlangen-Nürnberg
Bismarckstrasse 1 1/2
D-91054 Erlangen
Germany

schmeisser@mi.uni-erlangen.de

Let f be an entire function of exponential type whose restriction to $\mathbb{R}$ belongs to $L^2(\mathbb{R})$. Then, for all $w \geq \sigma/\pi$ the following formula, usually named after Whittaker–Kotel'nikov–Shannon (WKS-formula for short), holds :

$$f(z) = \sum_{n\in\mathbb{Z}} f\left(\frac{n}{w}\right) \operatorname{sinc}(wz - n) \quad (z \in \mathbb{C}). \tag{1}$$

Here sinc is the *sinus cardinalis* defined by

$$\operatorname{sinc} z := \begin{cases} \frac{\sin(\pi z)}{\pi z} & \text{if} \quad z \in \mathbb{C}\backslash\{0\}, \\ 1 & \text{if} \quad z = 0. \end{cases}$$

This formula is of fundamental interest in signal processing since it says that a signal of finite energy bandlimited to the interval $[-\sigma, \sigma]$ can be reconstructed from the sequence of samples $\{f(n/w) : n \in \mathbb{Z}\}$. However, the practical use of this formula is limited since the truncated series (1) may not give a good approximation to f.

We present old and new modifications of (1) which are more suitable for practical purposes. Contributions to the question for a best approximation from a finite number of samples are also made.