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Growth in simple groups of Lie type

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Let G be a simple group of Lie type ; let K be a finite field. Let A be a subset of $G(K)$ that generates $G(K)$. Assume that $|A| <= |G(K)|^{1-\varepsilon}$, $\varepsilon > 0$, where $|S|$ denotes the number of elements of a set S.

In 2005, I proved that, for $G = \mathrm{SL}_2$, $K = F_p$, we always have

$$|A\ A\ A| \ll |A|^{1+\delta},$$

where $\delta > 0$ and the implied constant depend only on ε. In other words, any set of generators that has room to grow does grow rapidly.

Some time later, I generalised this result to $G = \mathrm{SL}_3$ and (**jointly with N. Gill**) to $G = \mathrm{SL}_n$ (in the case of small sets A). In a very recent development, Breuillard, Green and Tao, on the one hand, and Pyber and E. Szabo, on the other, have announced general statements of the theorem valid for all finite simple groups of Lie type. We will discuss the program leading to the proofs of all of these results.