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Properties of sum and product sets in an arbitrary finite fields

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Let us denote as $\mathbb{F}_q$ a finite field with $q = p^r$ elements. The following problem will be discussed : given a natural $n \geqslant 2$, a real $\varepsilon \in (0, 1)$ and any subset $A \subseteq \mathbb{F}_q$, not contained in any subset $\{ds : s \in S\}$, where $d \in \mathbb{F}_q$ and S is an arbitrary proper subfield of $\mathbb{F}_q$, satisfying $|A| > q^{\frac{1}{n-\varepsilon}}$. Is there exist a natural $m = m(n)$ for which one can find a natural $N = N(n, r, \varepsilon)$ such that an arbitrary element $\mathbb{F}_q$ can be represented in the form $x = x_1 + x_2 + \ldots + x_N$, with $x_i \in \{a_1 \cdot a_2 \cdot \ldots \cdot a_m : a_j \in A, j = 1, 2, \ldots, m\}, i = 1, \ldots, N$. It turns out that the value $m = 2n - 2$ is the solution of the this problem and an explicit upper bound for number N can be obtained. However, in some cases the value $m = n$ can be a solution of this problem. Several results of this type will be presented.