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## Sum-product theorems for polynomials

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Suppose  $A$  is a set of numbers and  $f(x, y)$  is a polynomial, how small can  $f(A, A)$  be? If  $f(x, y) = x + y$  or  $f(x, y) = xy$ , then  $f(A, A)$  can be very small indeed if  $A$  is a progression. However, Erdős and Szemerédi proved that  $A + A$  and  $AA$  cannot be simultaneously small when  $A$  is a set of real numbers.

In this talk, I will present several new results for other polynomial and rational functions  $f$  over finite fields.

*Joint work with Jacob Tsimerman.*