

# Actuarial Research Conference 2006 - Abstracts

## Conférence sur la recherche en actuariat 2006 - Résumés

(in alphabetical order of speakers's name (in bold))  
(en ordre alphabétique du nom du conférencier (en caractères gras))

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### INVITED TALK – CONFÉRENCE INVITÉE

#### On a Global Education and Examination System

**Jean-Louis Massé**

President, International Actuarial Association

**Abstract:** The International Actuarial Association is studying the possibility of creating a global education and examination system, ideally with the help of universities, to accelerate the growth of the profession and to answer the needs for professional actuaries in the world. The IAA hopes to take a decision soon on this matter. The president of the IAA will talk about the great challenges of this ambitious and promising project.

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### CONTRIBUTED TALKS – CONFÉRENCES CONTRIBUÉES

#### Assessing Alternative Financing Methods for the Canadian Health Care System in View of Population Aging

**Doug Andrews**

University of Waterloo, Canada

**Abstract:** The cost of the Canadian health care system is approximately 10% of Gross Domestic Product (GDP). Survey-evidence suggests that Canadians do not wish to have additional funds spent on health care but believe that the system should be able to deliver better quality. Due to declining fertility rates and increased life expectancy, the Canadian population is aging. Over the next 25 years, the dependency ratio will increase dramatically, primarily due to the baby boom generation reaching age 65. This will place twofold cost pressures on governments responsible for maintaining the health care system:

- As a consequence of increased life expectancy, on average, Canadians will have a longer period of health care consumption. Although age-specific cost may not increase, with an aging population aggregate annual health care expenditures are expected to increase.
- The dependency ratio is a proxy for the size of the labour force and hence for the GDP. The increasing dependency rate is likely to result in a slowdown in GDP growth.

In the first section, this paper attempts to quantify these factors. It develops a single measure combining cost and quality to demonstrate the magnitude of the challenge and to be used in assessing alternative financing models.

In the second section, this paper examines a number of different approaches to health care financing including user fees and alternative compensation methods for physicians. The paper highlights documented information on the success of these approaches in containing costs and improving quality where they have been applied in Canada or internationally. The paper evaluates the desirability of implementing these approaches in Canada using the assessment measure developed in the first section.

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#### Modelling and Estimating Individual and Firm Effects with Panel Data

**Jean-François Angers**, Denise Desjardins Georges Dionne and François Guertin  
Centre de recherche sur les transports, University of Montreal and HEC Montreal, Canada

**Abstract:** In this article, we propose a detailed analysis of the modelling and estimation of distributions of vehicle accidents. This analysis uses panel data to account simultaneously for individual as well as fleet effects. The distribution of accidents can be affected by both observable and non-observable factors. Non-observable factors are modelled as random factors.

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## Extreme Behavior of Multivariate Phase-Type Distributions

**Alexandru V. Asimit** and Bruce L. Jones  
The University of Western Ontario, London, Canada

**Abstract:** This paper investigates the limiting distributions of the componentwise maxima and minima of suitably normalized iid multivariate phase-type random vectors. In the case of maxima, a large parametric class of multivariate extreme value (MEV) distributions is obtained. The flexibility of this new class is exemplified in the bivariate setup. For minima, it is shown that the dependence structure of the Marshall-Olkin class arises in the limit.

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## Competing Risk Model for Corporate Default and Exit Analysis: Cox-Relative Hazard Model and Extension with Stochastic Frailties

Reg J. Kulperger and **Taehan Bae**  
The University of Western Ontario, London, Canada

**Abstract:** Publicly traded companies can leave a public system by corporate default or an exit due to merger. We consider a multivariate Cox relative hazard model and further extension to stochastic frailty. For U.S. Industrial firms spanning 1992 to 2004, quarterly firm specific financial variables and monthly macroeconomic variables are available. The intensity function in the Cox model is a function of these firm specific, macroeconomic covariates and some industry or sector specific covariates. The data requires cleaning and decisions on how to handle inconsistencies and combining different data sets. A maximum partial likelihood method is used to estimate the parameters. We consider some attempts to assess the out of sample behavior of the model. The basic model is extended to include stochastic frailty. Our extension allows both positive and negative correlation between industry sectors.

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## Optimal Retention Levels in Dynamic Reinsurance Markets

**Enrico Biffis**  
City University, UK

**Abstract:** We consider the problem of determining optimal retention levels for insurers willing to mitigate their risk exposure by purchasing proportional reinsurance. We revisit De Finetti's classical results in continuous-time and allow reinsurance premiums and retention levels to change dynamically in response to claims experience and market performance. We also take up some ideas from dynamic reinsurance markets to intertwine De Finetti's work and Markowitz's mean-variance portfolio theory.

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## Lundberg-type Approximations for Defective Renewal Equations: a Heavy-traffic Perspective

**Jose Blanchet**  
Harvard University, USA

**Abstract:** Renewal equations for which the renewal time distribution,  $F(\cdot)$ , is defective (that is  $F(\infty) < 1$ ) arise often in many applied probability settings, such as insurance risk theory, queueing theory, reliability and branching processes among others. The text by Willmot and Lin (2001) discusses a great variety of applications to insurance and connections between defective renewal equations (DRE's) and compound geometric sums. Our focus is on DRE's that are close to be proper, that is,  $F(\infty) \simeq 1$ . In the queueing context, this situation is often related to systems operating under heavy-traffic and it also arises naturally in risk theory applications that involve low net profit environments (for instance, low safety loading or small interest rates). Motivated by these applications, we develop asymptotic expansions for the solution of DRE's in powers of  $(1 - F(\infty))$ . In the presence of light-tailed claims, our expansions can be connected naturally to so-called Lundberg approximations (popular in the analysis of DRE's). However, our expansions can also be developed in the context of heavy-tailed claims; hence, we are able to develop Lundberg-type approximations in the presence heavy-tailed claims. Connections to more standard heavy-tailed asymptotics for ruin probabilities and applications to ruin models perturbed by a diffusion will also be discussed.

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## Multivariate Modeling of Asset Returns for Investment Guarantees Valuation

Mathieu Boudreault, HEC Montreal, Canada  
Christian-Marc Panneton, Industrial Alliance

**Abstract:** Stochastic models have been used for valuing the CTE provision of segregated fund guarantees in Canada since 2002. Most publications on this topic have used an univariate estimation technique and applied it to a multivariate context. In this talk, we will analyze some of these approaches and compare them with a true multivariate estimation using the Canadian, U.S., U.K. and Japanese markets. Such a multivariate estimation requires the modeling of both the volatility and correlation structures. We therefore discuss the use of the regime-switching model in a multivariate setting. Many issues need to be considered when using a Markovian environment for multiple indices such as how the regimes and transitions are defined. Numerous multivariate GARCH models are also considered (VECH, BEKK, CCC, FARCH), notably, Engle's DCC class of GARCH models, which provides for a 2-step consistent estimation of volatilities and correlations. This approach is very useful to represent the dynamics of large correlation structures. Copula techniques are also explored to build multivariate distributions. As a final exercise, we have conducted a Monte Carlo experiment in which we have supposed the data come from a multivariate stochastic volatility model having leverage and/or fat tails with known parameters. This helps determine how the CTE computed with either the multivariate GARCH or regime-switching models compare with the true CTE. Our analysis shows that the model selection process is not the same in a multivariate setting than in an univariate framework. With the four markets used in our analysis, we have come up with the following conclusions: (1) the constrained univariate regime-switching model used by many practitioners provides the worst fit to historical data; (2) the multivariate GARCH models provide a better fit than multivariate regime-switching models; (3) there is significant variability in the 10-year CTE provisions computed with the multivariate models. As a result, the most appropriate model will depend on the specific portfolio that needs to be modeled.

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## Equity Indexed Annuities: the Quest for Optimality

Phelim P. Boyle  
University of Waterloo, Canada

**Abstract:** The retail investor faces a bewildering array of structured investment products. These contracts often include a rich variety of embedded option features. Generally they combine downside protection in bear markets with the potential for upside appreciation in bull markets. Some have argued that these products expand the investor choice set while others contend that the profusion of product types only serves to confuse the consumer. These vehicles include the broad range of equity indexed annuities sold by life insurance companies and the wide variety of structured products sold by banks. This paper will survey some of the features of the current structured product landscape. There is evidence that that these products tend

to be costly, complex and inefficient. We discuss how the design of these instruments can be tailored to the individual investor's preferences and explore the possibilities of improving on the current design. The paper is based on joint work with my colleague, Weidong Tian of the University of Waterloo.

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## Random Switching Times Among Randomly Parameterized Regimes of Random Interest Rate Scenarios

**James G. Bridgeman**

University of Connecticut, Storrs, USA

**Abstract:** Most of the usual stochastic interest rate models were designed and calibrated to provide plausible behavior in expected value and variance, which are the raw material for first approximation pricing and hedging of financial instruments. Standards of practice and often regulators require actuaries to stress test financial positions over long projection horizons against extreme interest rate paths. The behavior of extreme paths in the usual stochastic interest rate models is not nearly so plausible as the behavior of their expected values and variances. This paper proposes a new class of models that deliver more plausible extreme paths while preserving the usual expected value and variance behavior.

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## The Risk Management of a DB Underpin Pension Plan

**Kai Chen**

University of Waterloo, Canada

**Abstract:** Hybrid pension plans offer employees the best features of both defined benefit and defined contribution plans. In this work, we consider a hybrid design offering a defined contribution benefit with a defined benefit guaranteed minimum underpin. We consider valuation and risk management for this hybrid pension plan. A hedging portfolio is constructed using inflation-linked bonds, and a stock index. The payoff of the guarantee of the hybrid pension plan is similar to the payoff of an exchange option, which was developed by Margrabe (1976). We separate the pension guarantee by month and implement delta hedging to each part. We also calculate the monthly hedging cost and test the sensitivity to the crediting rate and the contribution rate.

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## An Integro-differential equation for a Sparre Andersen Model with investments

**Corina D. Constantinescu** and Enrique A. Thomann

Oregon State University, Corvallis, USA

**Abstract:** This talk considers one of the classical problems in the actuarial mathematics literature, the collective risk model. The claim number process  $N(t)$  is assumed to be a renewal process, the resulting model being referred as the Sparre Andersen risk model. The inter-claim times form a sequence of independent identical distributed random variables with distribution Erlang( $n, \beta$ ). The additional non-classical feature is that the company invests in a risky asset with returns modeled by a diffusion. The analysis is focused on the probability of ruin  $\psi(u)$ , where  $u$  is the initial surplus. It is shown that  $\psi(u)$  satisfies a certain integro-differential equation. As an example the equation for an Erlang(2,  $\beta$ ) inter-arrival times distribution and a geometric Brownian motion for the returns from investments is considered and the asymptotic decay of the probability of ruin is investigated.

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## Strategic Valuation

**Steve Craighead**, Towers Perrin - Tillinghast Software Solutions

Greg Slone, Nationwide Insurance

**Abstract:** In the process of assigning interest-crediting rates for accumulation of net premium deposits, there is competition between insurance companies. One company may set their rates based on new money rates, whereas another may set theirs based upon current portfolio rates, or any combination of the two. The different companies also use different metrics to determine their rate-setting practices. These metrics may be designed to measure competitive information, expected customer behavior, business line and/or company profitability, or other possible indicators. The determination of a competitive advantage must also account for regulatory control as well as the economic impact on the competitive and regulatory environment. The modeling of competitive rate setting must address all of these issues. In our initial research, we attempted to implement the above strategic situation within a traditional game theoretic environment. However, due to the complexity of our industry, any attempt to recast the problem therein was frustrated. This has led us to develop the strategic valuation system Simian, which allows for the intricate inter-relationships required. We outline the fundamental C++ classes that allow us to model different rate-setting processes in the presence of competitors, regulators, and various states of the economy. We have found that these classes are very flexible and can be easily modified to account for an array of strategic questions.

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## Quality Control of Risk Measures: Backtesting Var Models

**Victor H. de la Peña**, Columbia University  
Ricardo Rivera, State of New York Banking Department and NYU  
Jesús Ruiz-Mata, Lehmann Brothers

**Abstract:** This paper introduces a statistical approach to assess the quality of risk measures (QCRM). The approach is applied to the problem of evaluating the accuracy of Value-at-Risk (VaR) models used to predict the maximum losses in a bank's portfolio with a given confidence level. The current value VaR backtesting method developed by the Basel Committee for Banking Supervision controls the probability of rejecting the VaR when it is correct. However, the test has limited power to distinguish an accurate model from an inaccurate one and its power is not an appropriate measure of the model's validity (the probability of accepting the model when it is correct). Using the same information and changing the role of the null and alternative hypotheses, QCRM exploits the binomial structure of the testing problem to: a) provide a uniform reduction (over Bassell's) of the probability of accepting the model when it is incorrect and b) provides a uniformly most powerful test which measures the probability of accepting the model when it is correct. These improvements are partially offset by a small increase in the probability of rejecting the model when it is correct.

The test is based on correct coverage one-sided confidence intervals for the probability of an exception using the technique of pivoting the cumulative distribution function. The test results in new acceptance and rejection regions that are a complementary standard to the current regulatory regions. We compare the proposed confidence intervals to the ones used in financial literature and show their comparative strength.

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## Bayesian Inference Resistant to Outliers, using Super Heavy-tailed Distributions, for the Calculation of Premiums

**Alain Desgagné**, Université du Québec à Montréal, Canada  
Jean-François Angers, Université de Montréal

**Abstract:** We assume that the claim sizes for several risks are conditionally independent random variables given the same scale parameter. The Bayesian predictive distribution of a next claim size is used to estimate the pure premium. Robust procedures to conflicting information (prior or outliers) depend mainly on the tail behavior of the likelihood and prior densities. For that purpose, the notion of left and right log-credence is introduced to characterize respectively the left and right tails of a density defined on the positive line. Simple conditions are established to determine the proportion of observations that can be rejected as outliers. It is shown that the posterior distribution converges in law to the posterior that would be obtained from the reduced sample, excluding the outliers, as they tend to 0 or infinity, at any given rate. An exemple of

calculation of a pure premium is given. We compare the log-normal model with the robust super heavy-tailed (log-Pareto type) distributions model.

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## Bounds on the Ruin Probability in a Controlled Risk Model

**Maikol Diasparra** and Rosario Romera  
Universidad Carlos III de Madrid, Spain

**Abstract:** We consider a discrete risk process modeled by a Markov Decision Process. The surplus could be invested in stock market assets. We adopt a realistic point of view and we let the investment return process to be statistically dependent over time. We assume that follows a Markov Chain model. To minimize the risk there is a possibility to reinsure a part or the whole reserve. We consider proportional reinsurance. Recursive and integral equations for the ruin probability are given. Generalized Lundberg inequalities for the ruin probabilities are derived. Stochastic optimal control theory is used to determine the optimal stationary policy which minimizes the ruin probability. To illustrate these results numerical examples are included considering claim distribution of the PH type.

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## The Theoretical Problem of Managing a fund by entering into independent bets

**Mathieu Dufour**  
Université du Québec à Montréal, Canada

**Abstract:** Let us define a bet (of an amount  $M$ ) as a random experiment where the gain is  $M$  with probability  $p$  and  $-M$  with probability  $1 - p$ . A necessary condition for a risk-averse person to enter this bet is that  $p > 0.50$ . Suppose that a fund manager can, at the end of each period, gamble a proportion  $x_i$  of his portfolio in a bet with  $p_i > 0.50$ , for a finite choice of bets  $i = 1, 2, \dots, n$ . It is easily seen that a strategy that aims to maximise the expected gain yields to ruin. However, there exists a strategy that maximises the expected return. Determining it, quickly becomes more complex as the number  $n$  of possible bets increases, especially if the outcomes are not independent. We shall study this problem, which has surprising links with Shannons's information theory.

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## Fitting Combinations of Exponentials to Probability Distributions

**Daniel Dufresne**  
The University of Melbourne, Australia.

**Abstract:** Two techniques are described for approximating distributions on the positive half-line by combinations of exponentials. One is based on Jacobi polynomial expansions, and the other on the logbeta distribution. The techniques are applied to some well-known distributions (degenerate, uniform, Pareto, lognormal and others). In theory the techniques yield sequences of combination of exponentials that always converge to the true distribution, but their numerical performance depends on the particular distribution being approximated. An error bound is given in the case of the logbeta approximations.

Some applications are:

**Risk theory.** It has been known for some time that the probability of ruin is simpler to compute if the distribution of the claims, or that of the inter-arrival times of claims, is rational. The simplifications which occur in risk theory have been well-known in the literature on random walks (see, for instance, the comments in Feller (1971)) and are also related to queueing theory.

**Convolutions.** The distribution of the sum of independent random variables with a lognormal or Pareto distribution is not known in simple form. Therefore, a possibility is to approximate the distribution of each

of the variables involved by a combination of exponentials, and then proceed with the convolution, which is a relatively straightforward affair with combinations of exponentials.

**Financial mathematics.** Suppose an amount of money is invested in the stock market and is used to pay an annuity to a pensioner. What is the probability that the fund will last until the pensioner dies? The answer to this question is known in closed form if the rates of return are lognormal and the lifetime of the pensioner is exponential; this is due to Yor (1992). In practice, however, the duration of human life is not exponentially distributed, and the distribution of the required stochastic life annuity can be found by expressing the future lifetime distribution as a combination of exponentials.

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## Pandemic Influenza Claims Risk in the U.S.

**Tom Edwalds**

Munich American Reassurance Company, Chicago, IL, USA

**Abstract:** The Society of Actuaries (SoA) is sponsoring a study of the potential effect on U.S. life and health insurers of an influenza pandemic. Will insurers be able to pay all the claims if a new deadly and transmissible strain of the influenza virus appears? We will explain the model developed for the SoA by Jim Toole, FSA, of MBA Actuaries, and discuss the results of this model for a severe pandemic scenario.

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## Multivariate Phase Variables and Recursion Principles

**Karl-Theodor Eisele**

Université Louis Pasteur, Strasbourg, France

**Abstract:** We present discrete and continuous multivariate phase variables. We show the rationality of their generating functions, resp. their Laplace transforms and calculate the moments in the bivariate case. Finally, a recursion principle is given for compound multivariate phase variables. Examples, calculated by a VB-program, demonstrate how the results are easily applicable.

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## Ascertainment Bias in Estimating Rates of Early Onset Alzheimers's Disease: a Critical Illness Insurance Application

Angus Macdonald, **Carolina Espinosa**

Heriot-Watt University, Edinburgh, UK

**Abstract:** Estimation of rates of onset of rare, late-onset dominantly inherited genetic disorders is complicated by: (a) probable ascertainment bias resulting from the recruitment of strongly affected families into studies; and (b) inability to identify the true at risk population of mutation carriers. To deal with the latter, Gui and Macdonald (2002a) proposed a non-parametric (Nelson-Aalen) estimate  $\hat{\Lambda}(x)$  of a simple function  $\Lambda(x)$  of the rate of onset at age  $x$ .  $\Lambda(x)$  had a finite bound, which was an increasing function of the probability  $p$  that a child of an affected parent inherits the mutation, unfortunately the estimation procedure fails if  $\hat{\Lambda}(x)$  exceeds this bound, which can happen at quite low ages. We show that such failure may in fact be a useful measure of ascertainment bias. Usually we would assume  $p = 1/2$ , but in the presence of ascertainment bias  $p > 1/2$  in the sample, so the maximum value attained by  $\hat{\Lambda}(x)$  allows us to estimate  $p$ , and therefore the degree of ascertainment bias that may be present, leading to bias-corrected estimates of rates of onset. We apply these to early-onset Alzheimer's disease associated with mutations in the Presenilin-1 gene. We apply our new estimates to the actuarial questions of extra premiums, given genetic information, and adverse selection, if genetic information need not be disclosed to insurers writing critical illness (CI) insurance. CI insurance premium increases are in the main high. The possible cost of adverse selection in respect of PSEN-1 gene mutations under various moratoria appears to be negligible except in the case of small markets and severe adverse selection.

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## Decomposing Loan Portfolio Value-at-Risk and Expected Shortfall

**Tingting Fan** and Zhongfei Li  
University of Waterloo, Canada  
and Lingnan College (University), Sun Yat-Sen University, Guangzhou, PRC

**Abstract:** A variety of models have been proposed with the objective of calculating the loss distribution of a credit portfolio. From the loss distribution, one can obtain useful risk measures such as the Values-at-Risk (VaR) and Expected Shortfall (ES). While these risk measures are important, it is also of significant interest of obtaining information about attributing fair VaR contributions and ES contributions to loans underlying the portfolio.

This paper presents a new and efficient tool of decomposing loan portfolio's VaR and ES using the probability generating function corresponding to the portfolio loss distribution. Our objective is to assign appropriate risk contributions to the parts of a loan portfolio with dependent default risks. We consider a factor copula approach which has the capability of modeling the default dependent structure while maintaining a balance between tractability and parsimony. Using our proposed approach, we derive semi-explicit expressions of credit risk contributions to VaR and ES, which in turn allow us to calculate the risk contributions quickly and exactly. We illustrate our proposed technique via some numerical examples.

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## Application of Epidemiological Models in Actuarial Mathematics

**Runhuan Feng**, University of Waterloo, Canada  
Jose Garrido, Concordia University, Canada

**Abstract:** The emergence of the worldwide SARS epidemic in 2003 led to a revived interest in the study of infectious diseases. Mathematical models have become important tools in analyzing transmission dynamics and measuring effectiveness of controlling strategies. However, the research on this topic in actuarial literature has only gone so far as to set up epidemiological models which better reflect transmission dynamics of infectious diseases. In effort to build a bridge between epidemiological and actuarial modeling, we shall look into analyzing possible financial arrangements made against expenses resulted from medical treatments given to patients.

Based on classical compartment models, the first part of this paper is devoted to developing insurance policies for susceptible participants facing the risk of infection and then formulating financial obligations of both insurance parties using actuarial methodologies. For the purpose of practical application, the second part employs a variety of numerical methods for calculating premiums and reserves. In the end, the methods are demonstrated by designing insurance products for the Great Plague in Eyam and the SARS Epidemic in Hong Kong.

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## Robust Estimation of Generalized Additive Models in the Calculation of Mortality Tables

**Esteban Flores**, ITAM, México and SVS, Chile  
Ana Vernica Prez, IMSS, México

**Abstract:** Graduation theory is an important part of actuarial science, generally used by insurance companies as well as by worldwide supervising institutions. Such theory is a very useful tool to calculate mortality tables, therefore there is plenty of literature dealing with this problem since the beginning of last century. Graduation theory can be divided into two large groups of methods, parametric and non-parametric methods, which, apparently, are not connected. Among the parametric methods, it is worth outstanding linear models, non-linear models, generalized linear models and splines. While among the non-parametric methods,

we have moving-weighted-averages, Whittaker graduation and Kernel smoothing. Recent studies have put these methods together into the same context. The approach is based on a Generalized Linear Models (GLM) framework which includes non-parametric smoothing known as Generalized Additive Models (GAM). The main reason for using GAM is that they do not involve strong assumptions about the relationship implied in the standard parametric regression. However, when having outliers, estimations obtained by classical methods, such as maximum likelihood estimators, lose effectiveness. The classical theory of analysis of outliers says that we can analyze each one of the outliers looking for the feasibility to reduce its impact or to eliminate it. Under the assumption that all data is important in itself, specially when dealing with mortality, having outliers becomes a matter of particular concern. Thus, it is necessary to develop a robust model which can assure a stable graduation in presence of these outliers. Regularly, when calculating mortality tables, available data contains relevant outliers, and for that reason, they cannot be discarded when the mortality tables are calculated. Here we consider a new robust model of GAM, denoted by GAMR. The methodology used is based on the non-parametric estimation of minimum quadratic distance (QD) which has shown to be robust for linear models, non-linear models and GLM. Asymptotic properties of GAMR are also studied and a real mortality table is calculated.

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## Econometric Models for Interest Rates

R. Keith Freeland and Mary R. Hardy  
University of Waterloo, Canada

**Abstract:** In this paper, we fit numerous econometric models to short-term and long-term UK interest rates. Most of the models considered are regime switching models, where the distribution depends on a latent regime. These models are routinely used to capture the dynamics of financial returns. We compare both regime switching models and single state models. In particular we consider the CIR and GARCH models. In addition to considering Gaussian returns we also consider the more complex non-central chi-square distribution. The models are all estimated via maximum likelihood estimation. The models are evaluated based on their complexity, likelihood value and residuals.

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## A Multivariate Tweedie Family with Applications to Risk Measurement

Edward Furman and Zinoviy Landsman  
University of Haifa, Israel

**Abstract:** In the present talk we propose a multivariate extension of the well-known exponential dispersion models. We name this class the multivariate Tweedie family (MTwF) because its univariate marginal distributions correspond to the univariate Tweedie ones. Furthermore, the proposed family possesses a dependence structure, which is reflected by its covariance structure, and it allows good modelling multivariate portfolios with dependent claims or risks. As special cases, MTwF contains, for instance, the multivariate inverse Gaussian, multivariate gamma and multivariate compound Poisson distributions in the sense that their univariate marginals are inverse Gaussian, gamma and compound Poisson respectively. We demonstrate the construction method of MTwF, derive exact density functions of multivariate Tweedie random vectors and calculate higher order moments. Some useful Chebyshev's type inequities are also produced for easy evaluations of higher and lower probabilistic bounds. Lastly, some applications to risk measurement are considered.

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## An Alternative Approach to Calculation of IBNR Reserve in Health Insurance

Jinadasa Gamage, Illinois State University, Normal, IL  
Jed L. Linfield, Kaiser Permanente, Rockville, MD  
Krzysztof Ostaszewski, Illinois State University, Normal, IL

**Abstract:** Recent decades have brought about increased use of statistical methodologies in property/casualty reserving, as an alternative to the deterministic approach of the completion factor method. For the calculation of the incurred but not reported (IBNR) reserves in health insurance, practicing health actuaries often rely on a two-step process: first, they calculate completion factors or some other deterministic measure of IBNR, and then they complete the IBNR reserve calculation with a regression analysis or actuarial judgment on the most recent experience. In this work, we investigate the use of a statistical approach to the calculation of IBNR reserve directly, and in an integrated fashion, without an intermediate step of deriving the completion factors. We illustrate the methodology with a use of a realistic, but hypothetical data set, and derive confidence bounds for the IBNR estimate (with the use of a Taylor series approximation). We also investigate implications of the use of this, and possible non-parametric methodologies, on the actuarial reserving practice, especially in view of the Sarbanes-Oxley Act of 2002.

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## The Frequency and Amount of Inpatient and Outpatient Health Care Expenditures

Edward W. Frees, **Jie Gao**, and Marjorie A. Rosenberg  
University of Wisconsin-Madison, Madison, WI, USA

**Abstract:** In an effort to constrain resources spent on health care, modeling individual-level expenditures has become an important building block in cost management. This research intends to accomplish two purposes. One is to extend on the two-part model (TPM) by jointly modeling frequency counts of utilizations and expenditure amount per visit for both inpatient and outpatient care through a univariate aggregate loss model. The TPM is a dominant methodology in estimating health care demand where the first part models the probability of use and the second part models the total expenditures conditional on some use. TPMs provide a simplified modeling treatment of frequencies of care when number of utilizations are greater than one within a year and vary according to types of care. The second purpose is to propose a bivariate aggregate loss model based on interdependencies between inpatient and outpatient care. In the bivariate aggregate loss model, counts of outpatient visits are conditioned upon inpatient admissions and a random effects specification helps to explain correlations of expenditure amount within a subject. We demonstrate the method by examining public use data from the Medical Expenditure Panel Survey (MEPS) monitored by the US Agency of Healthcare Research and Quality. We find evidence in favor of the univariate aggregate loss model in modeling outpatient expenditures; bivariate aggregate loss models improve upon the univariate aggregate loss models for both inpatient and outpatient expenditures.

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## The Effects on the Funding and Contribution Variance using the Modified Spreading Model

Steven Haberman, M. Iqbal Owadally and **Denise Gómez**  
City University, London, UK

**Abstract:** The present work analyses the effects on the funding and the contribution rate when the only source of unpredictable experience is through volatile rates of return. The funding of a defined benefit scheme has been one of the main concerns of the sponsors of a pension plan. A volatile fund is not desirable, as it might imply volatile contributions. It is shown that the modified spreading model, developed by Owadally (2003), eliminates gains and losses, arisen by favourable or unfavourable experience, by paying a specific amount of unfunded liabilities through time. A comparison of the modified spreading model is made with the spreading model developed by Dufresne (1988). The modified spreading model is shown to be more efficient than the spreading model, as it minimises the variance of the fund and the contribution and as it leads to a smoother fund and contribution rate. Real investment rates of return of the pension fund, are assumed to be represented by two stochastic models: bootstrap sampling method by using historical data and the IID special case of the autoregressive model. The bootstrap sampling analysis considers two different assets, a high-risk asset given by UK equities and a low-risk asset given by gilts. Also, six different

asset allocations, three different periods of time to project the value of the fund, and three scenarios for the actuarial assumptions on the rates of return are considered and analysed. The basis of our work is found mainly in Owadally and Haberman (2004) and Owadally (2003).

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## actuar: an R package for Actuarial Science

Vincent Goulet

Université Laval, Québec, Canada

**Abstract:** R is a free software environment for statistical computing and graphics not unlike S-Plus. More than just another statistical environment, R is also a complete, vector based, programming language. For actuaries with an APL background, moving to R should be natural and they gain sophisticated statistical and graphical tools as a bonus. In the hope to promote the use of R in Actuarial Science, both in practice and in research, we started the actuar project in 2005. This consists in building a comprehensive package of actuarial functions for R. Here, we present the functions already in the package and forthcoming additions.

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## Optimal Mean-Variance Investment for an Insurer

Wenjing Guo

Nanjing University of Finance and Economics, PRC  
and University of Waterloo, Canada

**Abstract:** How to select the optimal investment strategy turns into a difficult problem faced by insurance corporations now. The traditional investment model considers that insurer invests all its surplus in risky assets. Anna finds that the ruin probability of insurance business with a risky investment equals 1. To reduce the ruin probability insurer should invests in both risky and non-risky asset and set part of its wealth aside to cope with random claims. In this paper, a finite interval investment of insurer is studied. The insurer is permitted to invest in risky assets and non-risky assets. The wealth of the insurer is divided into two parts according to a fixed proportion, one is retained to meet the random claims and the other is used to invest. Variance is applied to measure insurer's risk. The relationship between underwriting risk and investment risk is considered in the model. The optimal investment strategy is defined to minimize the investment risk and the insurance risk for a given expected eventual wealth. By solving the model, the explicit expressions of the optimal strategy and the efficient frontier are derived. In addition, the dynamical properties of the optimal strategy and the efficient frontier are discussed. We find that the amount invested in risky asset is positively relative to the insurer's expected eventual wealth and negatively relative to its safe load.

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## Mortality Improvement Scales for the Canadian Insured Lives

Johnny Siu-Hang Li, Mary Hardy and Ken Seng Tan

University of Waterloo, Canada

**Abstract:** In the valuation of products involving life contingencies, actuaries often rely on life tables with a forecast of future trends, typically summarized by multiplicative mortality improvement scales. In this study we derive mortality improvement scales for the Canadian insured lives by means of a joint model that consists of multiple fitting stages. In the first stage we project the mortality experience of the whole Canadian population using an extended version of the Lee-Carter model in which special attention is paid to the measure of uncertainty and to the possible recurrence of outliers. In the second stage we relate the experience of the insured lives and the whole population by the so-called Brass-type approach. We then summarize the projection by a small number of parameters to obtain the improvement scales. Finally, we discuss the effects of selection and smoker-status.

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## Extreme Value Analysis for Partitioned Insurance Losses

Ping-Hung Hsieh and **John B. Henry III**  
Oregon State University, Corvallis, USA

**Abstract:** Extreme value theory has become increasingly important in the actuarial literature for modeling the distribution of large claims in the situation when individual losses have been recorded. However, it is often the case that losses are available only in a partitioned form, i.e., that only the frequencies of losses residing in certain loss intervals are recorded. The objective of our project is to expand on our prior study of extreme partitioned data so as to offer a comprehensive extreme value analysis of losses in the partitioned data setting. By assuming only that the underlying survivor function of losses is regularly varying at infinity, we have derived a maximum likelihood estimator for the tail-index and investigated its performance through a simulation study. A necessary next step in this project is to propose a threshold selection procedure for determining a threshold above which the assumption of regular variation is valid. Estimators of important quantities such as expected loss above a high threshold, extreme quantiles, and tail probabilities will also be derived. We illustrate the proposed techniques in an example using actual partitioned loss data obtained from the Insurance Services Office.

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## Using Expert Opinion in Actuarial Science

**Michel Jacques** and Mathieu Pigeon  
Université Laval, Canada

**Abstract:** Actuaries may have to model costs for which data are not available, because they are either scarce or confidential. Third party liability coverage can be an example of such a situation. An alternative avenue to data collection, often used in engineering and policy analysis, is to ask human experts to make statements about the possible costs. The presentation will outline how this information could be used to obtain a cost distribution. Traditional ad hoc approaches to combine expert opinion will be described. Bayesian models will be reviewed and the all important subject of expert calibration will be discussed in detail. A calibrated model introduced by Mendel and Sheridan (1989) will be presented and implemented in the S language. We then consider an actuarial application by fitting a parametric distribution to the output of the calibrated model.

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## Entropy, Longevity and Annuities

**M. Khalaf-Allah**, S. Haberman and R. Verrall  
City University, UK

**Abstract:** In this paper the entropy measure applied in population biology by Demetrious (1976) is extended to measure the effect of any changes in the force of mortality on the cost of life annuity for different interest rate scenarios and levels of mortality improvements. This allows different sources of risk in a life annuity contract to be summarized in a one figure index. Numerical values for the entropy measure are derived using an approach which extends that of Keyfitz (1977). Results are illustrated using English life tables over the period from 1851 till 1991 and also by applying different mathematical models for mortality projections such as the Gompertz and the Sithole et al (2000) mortality projection models for both males and females aged 60 as an attempt to get a better understanding regarding the properties of the entropy measure. This is followed by testing the sensitivity of the results obtained with regard to the different factors that are likely to affect the value of the entropy measure. In this study the effect of gender, age, assumed interest rate and the level of mortality improvement are investigated.

## Quantifying and Correcting the Bias in Estimated Risk Measures

Joseph H. T. Kim and Mary Hardy  
University of Waterloo, Canada

**Abstract:** In this paper we explore the bias in the estimation of the Value at Risk and Conditional Tail Expectation risk measures using Monte Carlo simulation. We assess the use of bootstrap techniques to correct the bias for a number of different examples. In the case of the Conditional Tail Expectation, we show that application of the exact bootstrap can improve estimates, and we develop a practical guideline for assessing when to use the exact bootstrap.

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## Implementation of Arbitrage-free Discretization of Interest Rate Dynamics and Calibration of Swaptions and Caps in Excel VBA

Ohoe Kim and Swathi Gaddam  
Towson University, Towson, Maryland, USA

**Abstract:** We consider Libor market model and calibration process. We estimate the price of interest rate derivatives such as Caps and Swaptions. We use Monte Carlo methods with appropriate dynamics under spot measure and forward measure. Imposing martingale discretization property, the same quantities are estimated again to see a contrast or discrepancy in light of confidence interval. Based on algorithms discussed in the paper "Arbitrage-free discretizations of lognormal forward Libor and swap rate models" by Glasserman and Zhao (2000), the algorithms are implemented in excel VBA. Starting with current market data such as FRAs and Bond Prices, we describe steps involved to obtain the price of interest rate derivatives such as Caps and Swaptions, which would be beneficial to the practitioners in the industry.

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## On the Difference between Two Poisson Random Variables

Austin Lee and Nien-Chen Li  
SungKyunKwan University, South Korea and Boston University, USA

**Abstract:** With recent innovative developments in the theory and computational techniques for generalized linear models, statistical methods like Poisson regression become widely used in applications. For example, in actuarial science, the automobile insurance premium is closely tied into the number of accidents. Power functions are compared between the exact D-test and the test used in SAS PROC GENMOD in testing the difference between two Poisson means.

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## Threshold Life Tables and their Applications

Johnny Siu-Hang Li  
University of Waterloo, Canada

**Abstract:** The rapid emergence of super-centenarians has highlighted the importance of the tail of the survival distribution, and motivated researchers to look for alternative ways for closing off the life tables instead of the prevailing practice of using the value of one at an arbitrarily chosen age. Based on the asymptotic results in the extreme value theory, we propose a model - the threshold life table - that allows practitioners to extrapolate the survival distribution to the extreme ages and to determine the appropriate end point of the life table. The model is further extended to a dynamic version which takes account of the non-diversifiable longevity risk, which originates from the uncertainty in future trends. The theoretical results are finally applied to the stochastic valuation of a life annuity portfolio and to the prediction of highest attained age for various cohorts.

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## Broken-Heart Mortality

Lily Yunsui Li (with Mary Hardy)  
University of Waterloo, Canada

**Abstract:** Although actuaries often assume independence in joint life and last survivor insurance, it is generally accepted that mortality of married lives worsens after the death of their spouse. Earlier work proposed a Frank's copula to manage this dependence. In this work we propose a multiple state approach, which will provide a more direct measure of dependency. We will explore data on deaths by marital status and look at the impact of this model on annuity and insurance pricing.

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## Optimal Investment Strategy in a Discrete-Time Model with Regime-Switching and Uncertain Time-Horizon

Zhongfei Li+\* and Ken Seng Tan+  
+University of Waterloo, Canada  
\*Lingnan College (University), Sun Yat-Sen University, Guangzhou, PRC

**Abstract:** This paper considers the optimal investment problem of an investor who does not know with certainty the time of eventual exit. Under the market setting in which the underlying economy switches among a finite number of states and the short-selling of risky assets and the riskless asset is prohibited, and given the conditional distribution of the investor's time horizon, we establish a discrete-time model that maximizes the expected power-utility of the terminal wealth. The existence and the uniqueness of the optimal investment to the model is obtained, and an explicit expression of the optimal investment strategy is presented by using dynamic programming approach. Some properties of the optimal strategy are also examined.

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## Actuarial Judgment in the Calculation of Medical Liability

Jinadasa Gamage, Illinois State University, Normal, IL  
Jed L. Linfield, Kaiser Permanente, Rockville, MD  
Krzysztof Ostaszewski, Illinois State University, Normal, IL

**Abstract:** For the calculation of the incurred but not reported (IBNR) reserves in health insurance, practicing health actuaries often rely on a two-step process: first, they calculate completion factors or some other deterministic measure of IBNR, and then they complete the IBNR reserve calculation with a regression analysis or actuarial judgment, or both, on the most recent experience. In this work, we discuss areas where actuarial judgment needs to be used, even with the application of statistical methods for the calculation of IBNR. Actuarial judgment needs to be used since the prediction of IBNR for months with the most unpaid claims have the most uncertainty. We discuss the general issue of adding statistical methods to an actuarial analysis while maintaining actuarial judgment.

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## Policyholder Behavior Study in Variable Annuity with Guaranteed Minimum Withdrawal Benefit

Yan Liu  
University of Waterloo, Canada

**Abstract:** The Guaranteed Minimum Withdrawal Benefit (GMWB) guarantees a certain minimum monthly (quarterly) withdrawal amount to the policyholder equal to the total investment even if the account value reaches zero. Policyholder Behavior is hard to estimate and predict, but it's a critical part in pricing and hedging the Product. Factors affecting withdrawal and lapse decisions may include economic variables, in-the-moneyness of the benefit, personal situations (age, other income etc.), and product features (policy duration, benefit features). Assumptions here seem to be subjective since there is no published data. We will use a multinomial logistic regression model to estimate the effects of some of these factors. We model withdrawal and lapse behavior as discrete-time stochastic processes.

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## Some Optimal Dividend Problems in a Markov-modulated Risk Model

Shuanming Li, The University of Melbourne, Australia  
Yi Lu, Simon Fraser University, Canada

**Abstract:** In this paper, we derive some results on the dividend payments prior to ruin in a Markov-modulated risk model in which the claim inter-arrivals, claim sizes and premiums are influenced by an external Markovian process. A system of integro-differential equations with boundary conditions satisfied by the  $n$ -th moment of present value of the total dividends prior to ruin, given the initial environment state, is derived and solved. We show that both the probabilities that the surplus process attains a dividend barrier from the initial surplus without first falling below zero and the Laplace transforms of the time that the surplus process first hits a barrier without ruin occurring can be expressed in term of the solution of the above mentioned system of integro-differential equations. In the two-state model, explicit results are obtained when both claim amounts are exponentially distributed. Finally, a numerical comparison with the results obtained from the associated averaged compound Poisson risk model is also given.

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## Stochastic Analysis of Life Insurance Surplus

Natalia Lysenko  
Simon Fraser University, Canada

**Abstract:** The behaviour of insurance surplus over time for a portfolio of homogeneous life policies in an environment of stochastic mortality and rates of return is examined. We distinguish between stochastic and accounting surplus and derive their first two moments. A recursive formula is proposed for calculating the distribution function of the accounting surplus. We then examine the probability that the surplus remains positive in every insurance year. Numerical examples illustrate the results for portfolios of temporary and endowment life policies assuming an AR(1) process for the rates of return.

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## Estimation of the Cost of Credit Rating Downgrades

Andreas Milidonis and Shaun Wang  
Georgia State University, USA

**Abstract:** In this paper we plan to derive an analytical framework and to perform empirical estimations of the cost associated with credit rating downgrades, expressed as a function of distance-to-default. A company's credit-worthiness can be defined as its ability to meet future debt obligations. Consequently, the Cost of Downgrade can be defined as the reduction in the credit-worthiness of a company following a negative event (unfavorable news announcement, downgrade, default, or other loss). In essence, the occurrence of a negative event could initiate uncertainty about the company's creditworthiness, which uncertainty may be catalyzed by reduced expectations in growth and cash flows, the possibility of illiquidity, employees and customers leaving the company, decreased trade credit, restructuring costs and increased costs of borrowing capital. In the case of bankruptcy such scenarios could affect the recovery value of assets. Being able to quantify this

cost of downgrade across credit ratings and industries will allow us to refine the process of default in more detail than the current binary, "Default-No default" approach. Estimation of this cost function will shed light in the cost and allocation of capital across rating categories and industries. Furthermore, establishing a relationship between the cost of downgrade and credit spreads will imply useful results for credit risk managers.

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## Determinants of Group Health Insurance Demand

**Jorge Munoz Perez**, ING Mexico  
Tapen Sinha, ITAM, Mexico and University of Nottingham, UK

**Abstract:** There have been surprisingly few investigations regarding the factors that determine the demand for group health insurance. The first striking element of a group health plan is that the premium does not depend on the past claims by workers or dependents. We analyze the data of a company with 10,000 workers and their dependents. The data is linked to income of each worker. The results show that income level is a surprisingly important factor for claims. Specifically, claims rise with rising income. But, the ratio of claims and income falls with rising levels of income. We also investigate other factors such as the sex of the worker, the number and relationship with dependents, age and the form of payment. An important policy implication for the insurance companies offering such group health policies is not to ignore income distribution of the workers covered by the policy.

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## Pension Plan Evaluation using Conditional Tail Expectation (CTE)

**Claude Pichet**, René Delsanne and Carole Turcotte  
UQAM, Canada

**Abstract:** We present a new method to calculate the liabilities of a defined benefit pension plan. The interest rate used will be obtained through the calculation of the conditional tail expectation of the 20th percentile of the lowest returns of the simulated portfolios.

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## Application of Second-Order Cone Programming in Robust Portfolio Management

Phelim Boyle and **Ke Qiu**  
University of Waterloo, Canada

**Abstract:** Starting from the classical Markowitz model, we discuss how to incorporate parameter uncertainty into the mean-variance model. It is well known that the weights obtained in portfolio optimizers are too sensitive to the expected return vector. Using robust control theory, we construct a max-min model with uncertainty about expected returns and give the closed form solution. To solve the max-min model with no short sales and other more complicated constraints, we introduce Second-Order Cone Programming (SOCP)-a powerful tool in optimization and explain how to use Sedumi-a Matlab Package for SOCP. Finally, we indicate possible applications of SOCP in other robust portfolio management applications.

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## Method to Develop a Provision for Adverse Deviation (PAD) for the Longevity Risk for Impaired Lives

**Sudath Ranasinghe**  
University of Connecticut, Storrs, USA

**Abstract:** This paper examines longevity risk and its implications for impaired lives. Life annuities and other long-term living benefits are the most important insurance products concerned with longevity risk. There are no standard mortality tables for impaired annuitants, especially for those with chronic medical and acute conditions. Therefore, it is difficult to capture longevity risk using traditional mortality projection methods.

This study recognizes the main components of longevity risk and treats them separately to develop an analytical method to calculate the provision for adverse deviation (PAD). The study also considers implications of this PAD model to the other insurance products such as Life Settlement and Structured Settlements.

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## A Bias Reduction Technique for Monte Carlo Pricing of Early Exercise Options

Tyson L. Whitehead, Matt Davison and **R. Mark Reesor**  
The University of Western Ontario, London, Canada

**Abstract:** We present a new method for reducing the bias in Monte Carlo estimators of the price of American-style contingent claims. At each exercise opportunity (in a time discretization), we assume there is an unbiased estimator of the claim value at the next exercise opportunity. We approximate the distribution of this statistic using the Central Limit Theorem and use this to derive an expression for the bias. This expression is easily estimated in the context of a simulation, which allows for the straightforward computation of bias-reduced estimators of the claim value. We conclude by presenting a well-studied multivariate pricing example to show that this method offers significant improvements over the vanilla stochastic mesh technique and that it is a much more computationally efficient approach to reducing bias than nonparametric bootstrapping.

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## The Discounted Probability of Ruin in the Sparre Andersen Model

**Jiandong Ren**  
The University of Western Ontario, London, Canada

**Abstract:** In this paper, we consider the risk model with phase type inter claim times. We first derive a simple matrix form expression for the discounted joint probability of the surplus prior to ruin and the deficit at ruin when the initial surplus is zero. Then, we present a formula for the discounted probability of ruin, which is parallel to that for the probability of ruin in Asmussen (2000) when claim sizes are also phase type.

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## Explicit Martingale Representations for Brownian Functionals and Applications to Option Hedging

**Jean-François Renaud**, Université de Montréal, Canada  
Bruno Rémillard, HEC Montréal

**Abstract:** Using Clark-Ocone formula, explicit martingale representations for path-dependent Brownian functionals are computed. As direct consequences, the explicit martingale representations of the extremum of geometric Brownian motion and explicit hedging portfolios of path-dependent options are obtained.

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## Future Lifetime as a Fuzzy Random Variable

**Arnold F. Shapiro**  
Penn State University, University Park, PA, USA

**Abstract:** One of the key concepts in texts like Bowers et al (1997: 52) and Gerber (1997: 15) is the random variable (RV) future lifetime of a life aged  $x$ ,  $T(x)$ . In practice, of course, while age is an important factor in the determination of  $T(x)$ , there are other relevant factors. Moreover, some of the other dominant factors, like the state of health and the character of  $(x)$ , are fuzzy notions. This being the case,  $T(x)$  might more appropriately be written as  $T = T(x_1, x_2, \dots, x_n)$ , where the  $x_i$  s are features of  $(x)$ , and described as a fuzzy random variable (FRV). The purpose of this study is to explore this conceptualization.

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## Using Aggregated Log Returns to Speed-up Simulations

**Gurbakhsh Singh**

Simon Fraser University, Canada

**Abstract:** The financial guarantees embedded in insurance products are usually long term in nature and often very complex. Valuing such guarantees may require time-consuming simulations or approximations based on an estimated model for the rates of return. Calculation time can be reduced by simulating returns and other values of interest less frequently (e.g. yearly simulations instead of monthly). We study models for the aggregated log rates of return when the estimated model is a Brownian Motion, an AR(1) and a two-state regime switching. To illustrate the use of these aggregated models, conditional tail expectations of the guarantees provided by segregated funds with reset will be presented.

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## Heavy-Tailed Longitudinal Data Modeling Using Copulas

**Jiafeng Sun**, Edward W. Frees, Marjorie A. Rosenberg

University of Wisconsin-Madison, USA

**Abstract:** In this paper, we introduce longitudinal modeling in the framework of heavy-tailed data, that is, data where extreme values are likely to occur. Heavy-tailed data have been analyzed using cross-sectional regression models, where flexible distributions such as the generalized beta of the second kind, generalized gamma or Burr are used with functions of explanatory variables as parameters to handle cross-sectional behavior. The objective of this paper is to model the time-series behavior of heavy-tailed longitudinal data. Specifically, we use copulas to model the dependencies over time, and heavy-tailed distributions to model the margins. This method allows us to handle data with either positive or negative skewness, as well as heavy tails. A goodness-of-fit test of elliptical copulas is also introduced. To illustrate our method, Wisconsin nursing homes utilization data from 1995 to 2001 are analyzed.

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## Tail Behaviour of the Solution of a Random Difference Equation with Subexponential Innovations

**Qihe Tang**

The University of Iowa, Iowa City, IA

**Abstract:** We consider a random difference equation  $R \stackrel{D}{=} X + YR$ , where  $\stackrel{D}{=}$  denotes equality in distribution,  $X, Y$ , and  $R$  on the right-hand side are independent random variables. Under assumptions that  $X$  is subexponentially distributed on  $(-\infty, \infty)$  and that  $0 \leq Y \leq 1$  almost surely but not degenerate at 0 or 1, we obtain an asymptotic formula for the tail probability of the solution for this equation. Applications to actuarial science are proposed.

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## Fuzzy Volatility Forecasts and Fuzzy Option Values

**K. Rane Thiagarajah**

Illinois State University, Normal, USA

**Abstract:** Many financial time series, such as returns on stocks and foreign exchange rates, exhibit leptokurtosis and volatility varying in time. Decision-making problems, in general, are not well defined as their model parameters are not precisely known. As a result there has been growing interest in using fuzzy models in such cases. Fuzzy probability theory is a simple and potentially a useful way to propagate impreciseness through a cascade of calculations. The use of fuzzy probability theory, introduced by Buckley (2004) as a methodology for modeling and analyzing certain financial problems, is of particular interest to a number of researchers due to fuzzy probability theory's ability to quantitatively and qualitatively model those problems which involve vagueness and imprecision. In this paper, we summarize the basics of fuzzy random variables and introduce a class of fuzzy random coefficient (FRC) volatility models. Fuzzy option values and fuzzy forecasts are also discussed in some details.

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## Equity Return Model Selection

**Matthew C Till** (with Mary Hardy and Keith Freeland)  
University of Waterloo, Canada

**Abstract:** With equity-linked life insurance guarantees requiring sophisticated modeling of equity return processes, a number of different index models have been proposed in the public domain. Though the models are fit to monthly returns, the tails of resulting multi-year accumulation factors the models produce are of primary concern for investment guarantee modeling. This thesis presents some well known index return models, and fits them to the monthly-recorded S&P 500 index. It tests the goodness of fit of the models to the monthly data, and compares them using likelihood-based tests. Finally, an extension of common time series bootstrap procedures is devised to compare each of the models, with the emphasis placed on long term tail fits. The procedure is proposed as a tool for model selection purposes.

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## Impacts of Dynamic Credibility Premium Scheme, Deductible and Policy Limit on Ruin Probabilities

**Cary Chi-Liang Tsai** and Gary Parker  
Simon Fraser University, Canada

**Abstract:** This paper studies ruin probabilities based on the classical discrete time surplus process, in which the constant and varying premiums received in each period are considered, respectively. We apply the Bühlmann credibility theory to calculate the so-called Bühlmann credibility premium as the renewal pure premium received in each period. Effects of deductible and policy limit for the individual claim size random variables are also investigated. With the dynamic premium scheme, we want to study some problems like can the dynamic credibility premium scheme, deductible and policy limit significantly reduce the probability of ruin?

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## Short Period NonCatastrophic Rainfall Options

**Barry Turner**  
McGill University, Canada

**Abstract:** While a single severe storm with a return period of many years may cause extensive damage, the financial impact of more numerous noncatastrophic precipitation events is also significant. Many outdoor activities are adversely affected by even light rainfall, and can be sensitive to the timing of intermittent showers or thunderstorms at specific locations on scales as short as hours. However, financial products related to rainfall more commonly tied to a daily, monthly or seasonal index. We will explore the use of high-resolution rainfall measurements and modeling for the design and pricing of shortperiod rainfall options.

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## GB2 Regression with Insurance Claim Severities

Mitchell C. Wills\*, **Emiliano A. Valdez\*** and Edward W. (Jed) Frees+  
\*University of New South Wales, +University of Wisconsin - Madison

**Abstract:** While modeling the severity component of insurance losses is well documented in the actuarial literature, it is less frequent to find claims models where regressor variables are introduced. The standard norm as of late is to use Generalized Linear Models which offer the flexibility of being able to introduce regressor variables. In this paper, we offer the alternative procedure for fitting the family of Generalized Beta of the second kind (or GB2, for short) distributions in modeling insurance claim severities when there are concomitant variables. Based on four parameters, the GB2 family offers the flexibility of allowing different degrees of skewness and tail behavior for claim severities, and at the same time, this family of distributions recovers many commonly known long-tailed distributions. The concomitant variables play the important role of allowing to understand how they affect the tail behavior of the distribution of claim severities. For application and demonstration, we used a portfolio of automobile insurance contracts to show how to estimate the model parameters together with the regression coefficients. We further consider diagnostic tools and procedures for judging the quality of the goodness of the distribution fit to the empirical data. We find that the GB2 family outperforms several other models of insurance claim severities commonly found in the literature and/or used in practice.

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## Efficient Procedure for Valuing American Lookback Options

**Frank Xuyan Wang**  
University of Waterloo, Canada

**Abstract:** In this presentation, we propose a new and efficient algorithm for pricing American lookback options under the binomial lattice framework. The key to the proposed algorithm is to recognize that by extending one time step in the binomial model, the change to the extremum of the underlying state variable follows a specific pattern. By exploiting this, we are able to devise a recursive algorithm which enables us to compute the prices of American lookback options efficiently. The proposed method is simple and its relative effectiveness is assessed by comparing to other existence methods.

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## Optimal Reinsurance under VaR and CTE Risk Measures

**Chengguo Weng\***, Zhang Yi+, Jun Cai\*, Ken Seng Tan\*  
\*University of Waterloo, Canada  
+Zhejiang University, Hangzhou, PRC

**Abstract:** Let  $X$  be the loss to be incurred by an insurer. To protect against a potential large loss, an insurer would use a reinsurance arrangement. In doing so, the insurer cedes part of her loss, say  $f(X)$ , to a reinsurer, and thus the insurer retains a loss  $I_f(X) = X - f(X)$ , where the function  $f(x)$  is called a ceded loss function and the function  $I_f(x) = x - f(x)$  a retained loss function. Meanwhile, the insurer pays a reinsurance premium, say  $\delta_f(X)$ , to the reinsurer. Hence, the total cost or the total risk for the insurer is the sum of the retained loss and the reinsurance premium.

In this paper, we consider a class of increasing convex ceded loss functions. Motivated by Cai and Tan (2005) and based on a result of Müller and Stoyan (2002) on increasing convex functions, we derive the optimal ceded loss functions in the class, which minimize the value at risk (VaR) and the conditional tail expectation CTE of an insurer's total cost under the expectation premium principle. The results show that depending on the confidence level for VaR and the safety loading for the reinsurance premium, a stop-loss reinsurance is optimal in some cases while a quota-share reinsurance or the combination of a stop-loss reinsurance and a quota-share reinsurance is optimal in some other cases.

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## Credibility Theory for Generalized Linear Models

José Garrido and **Jun Zhou**  
Concordia University, Montreal, Canada

**Abstract:** Generalized linear models (GLM) are gaining popularity as a statistical analysis method for insurance data. For segmented portfolios, as in car insurance, the question of credibility arises naturally; how many observations are needed in a risk class before the GLM estimator can be considered credible? In this paper we study the limited fluctuation credibility of the results given by GLM as well as the more general case of generalized linear mixed model (GLMM). We will show how credibility depends on the sample size and the distribution of covariates. This provides a mechanism to obtain confidence interval for the estimates in GLM and GLMM.

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## Theory of Levy Processes Applied to Risk Models

**Xiaowen Zhou**  
Concordia University, Montreal, Canada

**Abstract:** In this talk we will go over some known results on spectrally negative Levy processes. Such results can be applied to the study of risk models. In particular, we could recover expressions of Gerber-Shiu functions for various risk models. Other applications will also be discussed.

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## Spatial and Longitudinal Analyses of Water Consumption and Obesity

Lan Mu and **Yanyun Zhu**  
University of Illinois at Urbana-Champaign, USA

**Abstract:** Water is essential for life. The Nationwide Food Consumption Surveys indicate that a portion of the US population is chronically dehydrated. Some research suggests that water intake may determine food intake and dehydration contributes to the development of obesity: today about 30% of the adult population and 16% of children are overweight or obese. Obesity causes all kinds of health problems, such as heart disease, and economic problems for both patients and society as a whole due to reduced productivity and increased health care costs. In this project we will use spatial and panel data analysis to investigate the issues related to water consumption and obesity.

Clarifying those issues will help individuals to live healthier and more productive lives, and reduce health care costs.

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## WORKSHOP – ATELIER

### Report on a Future Education Model for Canada

**Josephine Marks**  
Chair, Future Education Model task force of the Canadian Institute of Actuaries

#### Objectives of the workshop

The CIA would like to present and discuss a new approach to the education and qualification of actuaries in Canada. We hope that most academics involved in actuarial science education in Canada will participate in the workshop.

Length: 2 hours – Durée: 2 heures