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The least core value of schedule planning games

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Abstract

A schedule planning game is a cooperative game (N, v) where the cost $v(S)$ to a coalition S is the minimum sum of weighted completion times of jobs in S on one machine. Since it is always cheaper for an individual to schedule his or her job alone on one machine, cooperation in this context is difficult to achieve. The least core value of a cooperative game is the minimum penalty for defection that encourages all agents to cooperate. We show that computing the least core value of schedule planning games is NP-hard, and provide an approximation algorithm based on an oracle that approximately solves a maximally violated constraint problem. We show that any approximation algorithm for the maximum cut problem on undirected graphs can be used as such an oracle. In addition, we provide a fully polynomial time approximation scheme for this maximally violated constraint problem. Finally, we extend the analysis to the more general case where v is supermodular, and provide a class of optimization problems that includes various multiple machine scheduling problems and yields supermodular functions.

Joint work with Nelson Uhan.