Approximation Algorithms
June 12–14, 2006

Approximation algorithms for graph homomorphism problems

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Abstract

We introduce the maximum graph homomorphism (MGH) problem: given a graph G, and a target graph H, find a mapping $\varphi\colon V(G) \mapsto V(H)$ that maximizes the number of edges of G that are mapped to edges of H. This problem encodes various fundamental NP-hard problems including MAX CUT and MAX k-CUT. We also consider the following multiway uncut problem. We are given a graph G and a set of terminals $T \subset V(G)$. We want to partition $V(G)$ into $|T|$ parts, each containing exactly one element of T, so as to maximize the number of edges of G whose both endpoints lie in the same part. Multiway uncut can be viewed as a special case of *prelabeled* MGH where the input also specifies a prelabeling $\varphi'\colon U \mapsto V(H)$, $U \subset V(G)$, and the output has to be an extension of φ'. In multiway uncut, H consists of $|T|$ disconnected self-loops and $\varphi'\colon T \mapsto V(H)$ is a bijection.

Both the maximum graph homomorphism and multiway uncut problems have a trivial 0.5-approximation algorithm. For multiway uncut, we present a 0.8535 approximation algorithm based on a natural linear programming relaxation. This relaxation has an integrality gap of $\frac{6}{7} \simeq 0.8571$, showing that our guarantee is almost tight. For maximum graph homomorphism, we show that any constant improvement over the ratio of 0.5 implies an algorithm for distinguishing between certain average-case instances of the *subgraph isomorphism* problem that appear to be hard. Complementing this, we give a $\big(0.5+\Omega(\frac{1}{|H|\log|H|})\big)$-approximation algorithm, which gives an improvement for any fixed H.

Joint work with Michael Langberg and Chaitanya Swamy.