Approximation Algorithms
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On Steiner rooted-orientations of graphs and hypergraphs

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Abstract

Given an undirected hypergraph and a subset of vertices S with a specified root vertex r in S, the Steiner Rooted-Orientation problem is to find an orientation of all the hyperedges so that in the resulting directed hypergraph the "connectivity" from the root r to the vertices in S is maximized.

This is motivated by a multicasting problem in undirected networks as well as a generalization of some fundamental problems in graph theory. Our main results are the following approximate min-max relations:

- Given an undirected hypergraph H, if S is $2k$-hyperedge-connected in H, then H has a Steiner rooted k-hyperarc-connected orientation.

- Given an undirected graph G, if S is $2k$-element-connected in G, then G has a Steiner rooted k-element-connected orientation.

Both are optimal in terms of the connectivity bounds. These also imply the first polynomial time constant factor approximation algorithms for both problems. The proof is based on a new use of the submodular flow technique, and a decomposition technique used in the Steiner Tree Packing problem.

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