Approximation Algorithms
June 12–14, 2006

A fresh look at Steiner trees: Greedy vs primal-dual algorithms

Jochen Konemann
jochen@math.uwaterloo.ca
Department of Combinatorics & Optimization
University of Waterloo
200 *University Avenue West*
Waterloo, Ontario $N2L$ $3G1$
Canada

Abstract

The Steiner tree problem is a classical NP-hard optimization problem with a wide range of practical applications. In an instance of this problem, we are given an undirected graph $G = (V, E)$, a subset R of V of terminals, and nonnegative costs c_e for all edges e in E. The goal is to find a minimum-cost tree T in G that spans all terminals in R.

The best known approximation algorithm known for the Steiner tree problem is due to Robins and Zelikovsky (SIAM J. Discrete Math., 2005) and achieves a performance ration of 1.55. Robins and Zelikovsky's algorithm is a greedy algorithm. The best known LP-based algorithm for general graphs is a 2-approximation due to Agrawal, Klein and Ravi (SIAM J. Computing, 1995). The analysis of this algorithm is tight as the underlying undirected cut relaxation for the Steiner tree problem has an integrality gap of nearly 2. Rajagopalan and Vazirani (SODA, 2000) have recently proposed a new primal-dual $(1.5 + \varepsilon)$-approximation algorithm for the Steiner tree problem in quasi-bipartite graphs; these are graphs in which no two Steiner vertices are connected by an edge. Their algorithm is based on the so called bidirected cut relaxation for the Steiner tree problem.

Motivated by the result of Rajagopalan and Vazirani, most recent efforts on finding better LP-based approximation algorithms have focused on the bidirected cut relaxation. In this talk, we propose a new undirected formulation, and we first show that Khang and Robins' well-known 1-Steiner heuristic returns a 1.5-approximate Steiner tree with respect to our relaxation. Furthermore, we can show that the above analysis for the quasi-bipartite special case extends to general graphs; Robins' and Zelikovsky's 1.55-approximation can be interpreted as a primal-dual algorithm using our relaxation.

This is joint work with Kunlun Tan.