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Maximizing submodular set functions revisited

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Abstract

We consider the problem of maximizing a nonnegative monotone submodular set function $f\colon 2^N -> R+$, subject to independence constraint(s). A simple special case is when the constraint is to pick a subset of N of cardinality at most k. This already captures the well known maximum coverage problem and a tight $1 - 1/e$ approximation is obtained by the greedy algorithm. It is also known that the greedy algorithm has an approximation ratio of $1/(p+1)$ for a p-independence system (an example is a system obtained as an intersection of p matroids). This latter result (due to Fisher, Nemhauser and Wolsey 1978) does not seem to be as well known and special cases of it are often implicitly rediscovered. In this talk we revisit this work and give some illustrative applications, in particular to the generalized assignment problem and repairman problems. We then consider in more detail the case of $p = 1$, that is a single matroid constraint. A natural LP relaxation, proposed earlier, might have an integrality gap of $1 - 1/e$ which would improve the $\frac{1}{2}$ approximation provided by the greedy algorithm. We justify this optimism by proving it for a class of submodular functions that includes generalizations of maximum coverage. We use the so-called pipage rounding technique of Ageev and Sviridenko for this purpose.

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