The Hodge theory of algebraic maps

Mark de Cataldo

Department of Mathematics
SUNY at Stony Brook
Stony Brook, NY 11794
U.S.A.

Abstract

I will discuss recent results in joint work with L. Migliorini at Bologna. I will try to give a flavor of the reuslts which are concerned with the structure induced on the cohomology H^* of a projective manifold X by a projective map $f \colon X \to Y$. In particular,

- we decompose H^* as a double direct sum of Hodge structures polarized by the intersection form on X (this generalizes the Primitive Lefschetz Decomposition and the Hodge-Riemann Bilinear Relations),

- we determine the refined intersection form on the homology of the fibers of the map f (this generalizes Grauert-Mumford criterion for the contraction of curves on surfaces).

We show that these results imply directly a refined version of the Decomposition Theorem of Beilinson, Bernstein, Deligne and Gabber for arbitrary proper algebraic maps.