A geometric Littlewood-Richardson rule

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Abstract

I will describe an explicit geometric Littlewood-Richardson rule, interpreted as deforming the intersection of two Schubert varieties so that they break into Schubert varieties. There are no restrictions on the base field, and all multiplicities arising are 1; this is important for applications. This rule should be seen as a generalization of Pieri's rule to arbitrary Schubert classes, by way of explicit homotopies. It has a straightforward bijection to other Littlewood-Richardson rules, such as tableaux and Knutson and Tao's puzzles.

This gives the first geometric proof and interpretation of the Littlewood-Richardson rule. It has a host of geometric consequences, which I may describe, time permitting. The rule also has an interpretation in K-theory, suggested by Buch, which gives an extension of puzzles to K-theory, and in fact a Littlewood-Richardson rule in equivariant K-theory (ongoing work with Knutson). The rule suggests a natural approach to the open question of finding a Littlewood-Richardson rule for the flag variety, leading to a conjecture, shown to be true up to dimension 5. Finally, the rule suggests approaches to similar open problems, such as Littlewood-Richardson rules for the symplectic Grassmannian and two-flag varieties.