

Rational Lagrangian, Wall crossing of Floer homology and Homological mirror symmetry

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Abstract

This is a report of a project to prove homological mirror symmetry conjecture by M.Kontsevitch. I am working on it from several different points of view. One is transcendental one based on Asymptotic analysis. But in this talk I would like to explain more algebraic approach. Suppose we are given a symplectic manifold which is a total space of (singular) Lagrangian torus fibration. Then assuming the symplectic form is integral, there are finitely many Lagrangian submanifold which are Bohr Sommerfeld orbit and are fibers. Each such Lagrangian submanifold defines a moduli space \mathbb{C}_*^n which are supposed to corresponds to a irreducible component of a singular Calabi Yau manifold (which is a large complex structure limit). Given another “rational” Lagrangian submanifold, family of coFloer homologies parametrized by this family \mathbb{C}_*^n (which corresponds to the moduli space of bounding chains as was constructed in a joint work with Oh Ohta Ono) gives a holomorphic vector bundle (or more precisely object of derived category of coherent sheaves) on the infinitesimal(formal) neighborhood on each of such a component. We need to glue them by studying wall crossing of Floer homology from one BS orbit to another. In case some BS orbit is a singular fiber, it seems that one need to replace \mathbb{C}_*^n by something else which is not yet analysed enough.