Degenerations in the additive categories of almost cyclic coherent Auslander-Reiten components

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Let A be a finite dimensional algebra over an algebraically closed field k and $\mathrm{mod}_A(d)$ the affine variety of d-dimensional A-modules. The general linear group $\mathrm{Gl}_d(k)$ acts on $\mathrm{mod}_A(d)$ by conjugation, and the orbits correspond to the isomorphism classes of d-dimensional modules. We denote by $\mathcal{O}(M)$ the $\mathrm{Gl}_d(k)$-orbit of a module M in $\mathrm{mod}_A(d)$. Then one says that a module N in $\mathrm{mod}_A(d)$ is a *degeneration* of a module M in $\mathrm{mod}_A(d)$ if N belongs to the Zariski closure $\overline{\mathcal{O}(M)}$ of $\mathcal{O}(M)$ in $\mathrm{mod}_A(d)$, and we denote this fact by $M \leq_{\mathrm{deg}} N$. Thus $\leq_{\mathrm{deg}}$ is a partial order on the set of isomorphism classes of A-modules of a given dimension. We consider also another partial order $\leq_{\mathrm{ext}}$ on the category $\mathrm{mod}\, A$ of finite dimensional A-modules defined as follows:

$\bullet M \leq_{\mathrm{ext}} N :\Leftrightarrow$ there are modules M_i, U_i, V_i and short exact sequences $0 \to U_i \to M_i \to V_i \to 0$ in $\mathrm{mod}\, A$ such that $M = M_1$, $M_{i+1} = U_i \oplus V_i$, $1 \leq i \leq s$, and $N = M_{s+1}$ for some natural number s.

For all modules M and N in $\mathrm{mod}_A(d)$, we have $M \leq_{\mathrm{ext}} N \Longrightarrow M \leq_{\mathrm{deg}} N$ but the converse implication is not true in general.

Recall that a connected component $\mathcal{C}$ of the Auslander-Reiten quiver Γ_A of A is called *generalized standard* if $\mathrm{rad}^\infty(X, Y) = 0$ for all modules X, Y in $\mathcal{C}$. Further, $\mathcal{C}$ is called *almost cyclic* if all but finitely many modules of $\mathcal{C}$ lie on oriented cycles (in $\mathcal{C}$). Moreover, $\mathcal{C}$ is called *coherent* if every projective module P in $\mathcal{C}$ is the starting module of an infinite sectional path and every injective module I in $\mathcal{C}$ is the ending module of an infinite sectional path.

The aim of the talk is to describe when the partial orders $\leq_{\mathrm{ext}}$ and $\leq_{\mathrm{deg}}$ coincide for all modules of the same dimension from the additive category $\mathrm{add}(\mathcal{C})$ of a generalized standard almost cyclic coherent component $\mathcal{C}$ in Γ_A.