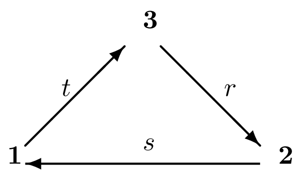
Otto Kerner, Cluster tilted algebras of rank three

Let K be an algebraically closed field, H be a finite dimensional connected hereditary K-algebra and $\mathcal{C}$ the cluster category of type H. If T is a squarefree tilting object in $\mathcal{C}$, then $\Gamma = \mathrm{End}_{\mathcal{C}}(T)$ is call a *cluster tilted algebra* of type H. One always can assume that T is the image of a tilting H-module in $\mathcal{C}$. If H contains at most 2 simple modules, up to isomorphisms, then Γ again is a hereditary algebra, derived equivalent to H.

If H is of rank 3 and $\Gamma = \mathrm{End}_{\mathcal{C}}(T)$ is not hereditary, then the quiver $\mathcal{C}$ of Γ has the following shape



with $r, s, t \geq 1$, The symbol $x \xrightarrow{m} y$ means that there are m arrows from x to y. Beineke, Brüstle and Hille showed, which of these cyclic quivers are the quivers of cluster tilted algebras.

If H has rank 3, then $r = s = t = 1$ if and only if H is of Dykin type (A_3) if and only if the Loewy length $\ell(\Gamma) = 2$. If H is representation infinite, then the Loewy length $\ell(\Gamma)$ depends on the number of regular direct summands of T:

Theorem. *Let H be representation infinite of rank 3 and $\Gamma = \mathrm{End}_{\mathcal{C}}(T)$ a cluster tilted algebra of type H, which is not hereditary. Then $\ell(\Gamma) = 3 + x$, where x denotes the number of regular indecomposable direct summands of T. Moreover, there is exactly one indecomposable projective Γ-module P with $\ell(P) = 3 + x$.*

The question is, whether it is practically possible to determine the Loewy length of Γ from the quiver $\mathcal{Q}$. It is easy to see, whether $x \leq 1$ holds. I will give in the talk an explicit combinatorial description for the cases $x = 2, 3$.

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