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Fixed points of endomorphisms over special confluent rewriting systems

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Gersten proved in the eighties that the fixed point subgroup Fixφ of an automorphism φ of a free group F_n is f.g. Cooper gave an alternative proof, proving also that the fixed points of the continuous extension of φ to the boundary of F_n is in some sense finitely generated. Such results were generalized to further classes of groups and endomorphisms in subsequent years.

On the other hand, Cassaigne and the author considered monoids defined by special confluent rewriting systems, generalizing some of the features of the F_n case. In fact, the undirected Cayley graph of these monoids is hyperbolic and has a quite convenient (compact) completion for the prefix metric : infinite reduced words. Uniformly continuous endomorphisms (algorithmically characterized) admit a continuous extension to the boundary.

In 2009, the author pursued this same approach to prove finite generation properties for both finite and infinite fixed points, centered on two properties of endomorphisms : boundary-injectivity and bounded length decrease. In the group case, the first property provides new proofs for already known results for monomorphisms of free groups and more generally free products of cyclic groups, as well as a new result on the infinite fixed points for monomorphisms of the latter.

However, all these results require uniform continuity for the prefix metric, i.e. injectivity in the group case. Thus it was a natural follow-up to consider arbitrary endomorphisms. Goldstein and Turner's beautiful proof for free group endomorphisms strongly requires group inversion, and the unique monoids of our type which embed into groups are free products of free monoids and cyclic groups. So free products of cyclic groups become the crucial case, and the proof can be generalized after overcoming some technical dificulties brought by finite order elements. We obtain thus a fully automata-theoretic proof of a result of Sykiotis, which may offer some insight into the algorithmic aspects of the problem. With respect to computability of Fixφ, we can generalize Maslakova's theorem to endomorphisms of $A^* * F_n$ whose restriction to F_n is an automorphism.

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