Combinatorics, topology and homological invariants of finite dimensional algebras

Benjamin Steinberg[*]

Finite dimensional algebras associated to various combinatorial structures such as real and complex hyperplane arrangements, oriented matroids and interval greedoids have been studied, first in connection to analyzing Markov chains. The unifying framework of LRB semigroup algebras was first proposed by Ken Brown. It turns out that LRB algebras are a rich class of split basic directed quasi-hereditary algebras properly containing the class of path algebras of acyclic quivers (e.g. split basic hereditary algebras).

In this talk we discuss how finite projective resolutions for the simple modules can be constructed from the actions of the LRBs on associated contractible simplicial complexes and hence the global dimension of these algebras correspond to homological properties of these simplicial complexes, like Leray numbers

For the case of what we call CW LRBs, which include real and complex hyperplane arrangements, oriented matroids and CAT(0) cube complexes, the corresponding algebras are Koszul with dual algebras that are incidence algebras. The minimal projective resolutions of the simple modules come from actions of the LRB on contractible CW complexes associated to these combinatorial structures.

[*]Department of Mathematics, City College of New York, Convent Ave at 138th Street, New York, NY 10031, USA.