Periodic symmetric algebras and triangulated surfaces

Andrzej Skowroński[*]

Let K be an algebraically closed field. By an algebra we mean a basic, indecomposable finite dimensional K-algebra, and we denote by mod A the category of finite dimensional right A-modules. Given a module M in mod A, its syzygy is defined to be the kernel $\Omega_A(M)$ of a minimal projective cover of M in mod A. A module M in mod A is said to be periodic if $\Omega_A^n(M) \cong M$ for some $n \geq 1$, and if so the minimal such n is called the period of M. An algebra A is defined to be periodic if it is periodic viewed as a module over the enveloping algebra $A^e = A^{\mathrm{op}} \otimes_K A$, or equivalently, as an A-A-bimodule. Periodicity of algebras is invariant under derived equivalence. We note that, if A is a periodic algebra, then A is selfinjective. Moreover, if A is periodic of period m, then for any indecomposable nonprojective module M in mod A the syzygy $\Omega_A^m(M)$ is isomorphic to M. It is conjectured that the following should be true

simple modules in mod A are periodic $\implies$ A is a periodic algebra.

This is known as the periodicity conjecture, and is an exciting open problem. It holds for example for the algebras of finite representation type (Dugas), representation-infinite algebras of polynomial growth (Białkowski, Erdmann, Skowroński), and blocks of group algebras of finite groups (Erdmann, Skowroński). The related interesting open problem is to describe all periodic algebras, up to Morita equivalence (respectively, derived equivalence). A. Dugas proved that an arbitrary selfinjective algebra of finite representation type, not simple, is periodic. The description of all representation-infinite tame periodic algebras of polynomial growth will be presented during the talk by J. Białkowski. It is also known that the preprojective algebras of Dynkin type, or more generally the deformed preprojective algebras of generalized Dynkin type, are periodic algebras, and are (with few small exceptions) of wild type.

During the talk we will present a complete description of all tame symmetric periodic algebras of period 4. We will show that these are exactly the tame symmetric algebras for

[*]Faculty of Mathematics and Computer Science, Nicolaus Copernicus University, Chopina 12/18, Torun, Kujawsko-Pomorskie 87-100, POLAND.

which all simple modules are periodic of period 4. Two prominent classes of such algebras were described 25 years ago: the algebras of quaternion type (Erdmann) and the trivial extensions of tubular algebras of type $(2, 2, 2, 2)$ (Nehring, Skowroński). But the fact that they are periodic of period 4 was proved only recently. The remaining tame symmetric periodic algebras of period 4 are related with algebras coming from triangulated surfaces. Recently, based on the work by Fomin, Shapiro and Thurston, the quivers of finite mutation type were classified by Felikson, Shapiro and Tumarkin. A prominent class of quivers of finite mutation type is given by triangulated surfaces with empty boundary and punctures. The Jacobian algebras of these quivers with respect to the potential defined by Labardini—Fragoso are tame symmetric algebras whose all nonprojective indecomposable modules are periodic of period 4 (Ladkani, Valdivieso—Diaz). It follows from our main result that these algebras are in fact periodic algebras of period 4. In order to make my talk more attractive, I mention that these Jacobian algebras form only a narrow subclass of the missing class of tame symmetric periodic algebras of period 4.

Joint work with K. Erdmann.