

Tubes in derived categories and cyclotomic factors of the Coxeter polynomial of an algebra

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Let Λ be a k -algebra of finite global dimension. We study tubular families in the Auslander—Reiten quiver of the bounded derived category $D^b(\Lambda)$ satisfying certain natural axioms (or more general, objects in $D^b(\Lambda)$ with periodic classes in the Grothendieck group $K_0(D^b(\Lambda)) \simeq K_0(\Lambda)$). We describe their influence on the cyclotomic factors of the Coxeter polynomial χ_Λ of Λ and discuss several numerical limitations for their possible shapes. Moreover, we show that our results provide an alternative, relatively simple proof (of number theoretical flavour) of non-trivial classical facts concerning tubular families in module categories, and also extend them slightly.

In particular, we give an easy to verify by hand (or by computer), necessary conditions for the existence of certain tubular families both in derived and module categories. We emphasize the “numerical” and “linear algebraic” reasons for common properties of tubular families appearing in important categories. We illustrate the results with several concrete examples and some applications.

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