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The two-flag problem for nilpotent operators, and the role of the orbit-pad

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My talk is on *joint work with D. Kussin and H. Meltzer*, see [1], [2]. It deals with the category of (graded) nilpotent operators, equipped with two finite flags of invariant subspaces. This problem is nontrivial even if the two flags degenerate to a single invariant subspace, a problem studied by C.M. Ringel and M. Schmidmeier in [1]. Actually the problem goes back—in the context of finite abelian groups—to G. Birkhoff in the 1930s. General aspects, have been been studied by various authors, among them D. Simson and P. Zhang.

The graded two-flag problem is contained in, and largely equivalent to, the problem to classify indecomposable vector bundles on a weighted projective line of triple weight type, with the weight data given by the two flag lengthes and the nilpotency degree. We show among other results that the two-flag problem for nilpotent operators yields an *almost Frobenius category* which is equivalent to a specific factor category of the category of vector bundles on the associated weighted projective line and whose attached stable category is triangulated, and then triangle-equivalent, to the stable category of vector bundles, a problem studied in [2].

We will discuss in some detail the tools from weighted projective lines entering in the proof, and in particular focus on the role of the *orbit-pad*, a combinatorial tool determined by the length- and nilpotency data. The orbit-pad carries simultaneously the structures of a finite abelian group, a poset and an action of the Picard group of the corresponding weighted projective line. We show that the orbit-pad determines the possible shapes of the Auslander—Reiten components for the two-flag problem, then allowing to determine a description in terms of dimension vectors for those components, containing a line bundle.

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References

[1] C. M. Ringel and M. Schmidmeier. Invariant subspaces of nilpotent linear operators. I. *J. Reine Angew. Math.*, 614:1--52, 2008.

[2] D. Kussin, H. Lenzing, and H. Meltzer. Nilpotent operators and weighted projective lines. *J. Reine Angew. Math.*, 685:33--71, 2013.

[3] D. Kussin, H. Lenzing, and H. Meltzer. The two-flag problem for nilpotent operators. In preparation.