

Mahler measure and the representation type of an algebra

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By a celebrated result of Kronecker, a monic integral polynomial p has Mahler measure $M(p) = 1$ if and only if p factorizes as product of cyclotomic polynomials. Rather little is known, however, about values of the Mahler measure near 1. In 1933, D. H. Lehmer found that the polynomial $T^{10} + T^9 - T^7 - T^6 - T^5 - T^4 - T^3 + T + 1$ has Mahler measure $\mu_0 = 1.176280 \dots$, and he asked if there exist any smaller values exceeding 1. In fact, the polynomial above is the Coxeter polynomial of the hereditary algebra with underlying graph.

In recent papers, we started to consider the largely unexplored topic of the Mahler measure $M(\chi_A)$ of the Coxeter polynomial χ_A of an algebra A . We say that an algebra A is of cyclotomic type if $M(\chi_A) = 1$ or of strongly cyclotomic type if A is accessible and all its convex subcategories are of cyclotomic type. There are many important examples of algebras of cyclotomic type. The following are new results:

Theorem. *Let A be a strongly simply connected algebra. Then A is of strongly cyclotomic type if and only if A is of tame representation type.*

Corollary 1. *Let A be a strongly simply connected algebra. Suppose that A is not of tame representation type, then $M(\chi_B) \geq \mu_0$ for some convex subcategory B of A .*

Corollary 2. *Let A be a strongly simply connected algebra. Suppose for every convex subcategory B of A with at most 10 vertices we have $M(\chi_B) = 1$ then A is tame. With the use of Galois coverings we can extend the above Theorem to further classes of algebras. We study the relation of the existence of cyclotomic factors of χ_A and the structure of the derived category of A .*

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