

# Varieties of invariant subspaces given by Littlewood–Richardson tableaux

Justyna Kosakowska<sup>\*</sup>

---

Given partitions  $\alpha, \beta, \gamma$ , the short exact sequences

$$0 \rightarrow N_\alpha \rightarrow N_\beta \rightarrow N_\gamma \rightarrow 0$$

of nilpotent linear operators of Jordan types  $\alpha, \beta, \gamma$ , respectively, define a constructible subset  $\mathbb{V}_{\alpha,\gamma}^\beta$  of an affine variety.

Geometrically, the varieties  $\mathbb{V}_{\alpha,\gamma}^\beta$  are of particular interest as they occur naturally and since they typically consist of several irreducible components. In fact, each Littlewood–Richardson tableau  $\Gamma$  of shape  $(\alpha, \beta, \gamma)$  contributes one irreducible component  $\overline{\mathbb{V}}_\Gamma$ .

We consider the partial order  $\Gamma \leq_{\text{bound}}^* \tilde{\Gamma}$  on LR-tableaux which is the transitive closure of the relation given by  $\mathbb{V}_{\tilde{\Gamma}} \cap \overline{\mathbb{V}}_\Gamma \neq \emptyset$ . In this paper we compare the boundary-relation with partial orders given by algebraic, combinatorial and geometric conditions. In the case where the parts of  $\alpha$  are at most two, all those partial orders are equivalent. We prove that those partial orders are also equivalent in the case where  $\beta \setminus \gamma$  is a horizontal and vertical strip. Moreover, we discuss how the orders differ in general.

*Joint work with Markus Schmidmeier from Florida Atlantic University.*

---

<sup>\*</sup>Faculty of Mathematics and Computer Sciences Nicolaus Copernicus University, Chopina 12/18, Toru, Kujawsko-Pomorskie 87-100, POLAND.