Conférence
« ARTA III : Avancées en théorie des représentations des algèbres »
16-20 juin 2014

Meeting
"ARTA III. Advances in Representation Theory of Algebras"
June 16–20, 2014

Integrations and computation of functor Ext

Mark Kleiner*

Let Λ be a finite dimensional algebra over a field K. If M, N are left Λ-modules, an *integration* from M to N is a K-linear map $f : \Lambda \underset{K}{\otimes} M \to N$ for which $f(\lambda_1\lambda_2 \otimes x) = \lambda_1 f(\lambda_2 \otimes x) + f(\lambda_1 \otimes \lambda_2 x)$. The reason for the name "integration" is that if one writes $\int(\int x\,\mathrm{dt})\mathrm{d}\lambda$ for $f(\lambda \otimes x)$ and assumes that λ_1, λ_2, and x are functions of the independent variable t, the above equation turns into the following valid formula from integral calculus:

$$\int\left(\int x\,\mathrm{dt}\right)\mathrm{d}(\lambda_1\lambda_2) = \lambda_1\int\left(\int x\,\mathrm{dt}\right)\mathrm{d}\lambda_2 + \int\left(\int \lambda_2 x\,\mathrm{dt}\right)\mathrm{d}\lambda_1.$$

There exists a *universal integration* ω_M from M, in that any integration from M to an arbitrary module factors uniquely through ω_M, and there exists a *couniversal integration* χ_N to N, in that any integration from an arbitrary module to N factors uniquely through χ_N.

Integrations suggest an alternative, simpler approach to the computation of the group $\mathrm{Ext}^1_\Lambda(C, A)$. We obtain an arbitrary s.e.s. $0 \to A \to B \to C \to 0$ from the split s.e.s. $0 \to A \to A \oplus C \to C \to 0$ by preserving the maps and twisting the module structure of $A \oplus C$ by an appropriate integration. These considerations shed new light on almost split sequences.

The notion of integration is inspired by the theory of bocses.

*Department of Mathematics, Syracuse University, Syracuse, NY 13244-1150, USA.