Joint AARMS-CRM Workshop: Recent Advances in Functional and Delay Differential Equations
1–5 November 2007

Modeling latency and relapse in diseases by delay differential equations

Xingfu Zou
Department of Applied Mathematics
University of Western Ontario
London, Ontario N6A 5B7
CANADA
xzou@uwo.ca

Abstract

A general mathematical model for a disease with an exposed (latent) period and relapse is proposed. Such a model is appropriate for tuberculosis, including bovine tuberculosis in cattle and wildlife, and for herpes. For this model with a general probability of remaining in the exposed class, the basic reproduction number $\mathcal{R}_0$ is identified and its threshold property is discussed. In particular, the disease free equilibrium is proved to be globally asymptotically stable if $\mathcal{R}_0 < 1$. If the probability of remaining in the exposed class is assumed to be negatively exponentially distributed, then $\mathcal{R}_0 = 1$ is a sharp threshold between disease extinction and endemic disease. A delay differential equation system is obtained if the probability function is assumed to be a step function. For this system, the endemic equilibrium is locally asymptotically stable if $\mathcal{R}_0 > 1$, and the disease is shown to be uniformly persistent with the infective population size either approaching or oscillating about the endemic level. Numerical simulations (for parameters appropriate for bovine tuberculosis in cattle) with $\mathcal{R}_0 > 1$ indicate that solutions tend to this endemic state.