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State-dependent delay, smoothness, and periodic solutions

Hans-Otto Walther
Mathematisches Institut
Universitaet Giessen
Arndtstr. 2
Giessen D 35392
GERMANY
Hans-Otto.Walther@math.uni-giessen.de

Abstract

Suppose a feedback system responds to the state $x = x(t)$ at time t only after a delay $d = d(x)$, at time $t+d$. A differential equation $x'(t) = f(x_t)$ for such a system contains a delay which is defined implicitly by the solution segment x_t. We study a scalar equation which models negative feedback with respect to the equilibrium at $x = 0$. Our main result is existence of a periodic solution when feedback is bounded and the equilibrium is linearly unstable. The proof employs differentiability of evolution operators on an infinite-dimensional Banach manifold X, closeness to a linear unstable subspace of the tangent space T_0X for local coordinates along small solutions, and the ejective fixed point principle.