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One-parameter localized traveling waves in nonlinear Schrödinger lattices

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Abstract

We address the existence of traveling single-humped localized solutions in the spatially discrete Schrödinger (NLS) equation. A mathematical technique is developed for analysis of persistence from a certain limit in which the dispersion relation of linear waves contains a triple zero. The technique is based on using the Implicit Function Theorem for solution of an appropriate differential advance-delay equation in exponentially weighted spaces. The resulting Melnikov calculation relies on a number of assumptions on the spectrum of the linearization around the pulse, which are checked numerically.

We apply the technique to the so-called Salerno model and the translationally invariant discrete NLS equation with a cubic nonlinearity. We show that the traveling solutions terminate in the Salerno model whereas they generally persist in the translationally invariant NLS lattice as a one-parameter family of solutions to the relevant differential advance-delay equation.