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Singularly perturbed state-dependent delay-differential equations

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Abstract

We study solutions of state-dependent delay-differential equations, in particular their asymptotic form in the singular perturbation case. Typical equations have the form

$$\varepsilon\dot{x}(t) = f\big(x(t), x(t-r)\big)$$

where $r = r\big(x(t)\big)$ for nonlinearities f and r, with ε small. More generally, equations with multiple delays may be considered. It is clear that such equations possess a very rich dynamical structure even for simple nonlinearities.

The"zeroth order" limit describes the gross shape of solutions, and at least in the case of a single delay for periodic solutions, can be treated with techniques of max-plus operators. Understanding the higher-order asymptotics of such solutions is a challenging and important problem, as it is intimately connected with questions of their existence, uniqueness, and stability. We shall describe recent work in this direction, based on dynamical systems techniques related to geometric singular perturbation theory, and motivated by numerical simulations.