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Localized waves in lattices of Fermi-pasta-Ulam type

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Abstract

This lecture emphasizes the use of center manifold reduction and normal form analysis for the search of spatially localized waves in lattices like the Fermi-Pasta-Ulam (FPU) lattice. Localized solutions include solitary waves (like in[2]) of permanent form, and travelling breathers which appear time periodic in a system of reference moving at constant velocity. We analyze the case when the breather period and the inverse velocity are commensurate in using the center manifold reduction method introduced by Iooss and Kirchgssner [1] in the case of travelling waves. This reduces the problem locally to a finite dimensional reversible differential equation. The normal form of the reduced system is integrable and admits solutions homoclinic to quasi-periodic orbits if a hardening condition on the interaction potential is satisfied. These orbits correspond to approximate travelling breather solutions superposed on a quasi-periodic oscillatory tail. The problem of their persistence for the full system (the normal form plus the remainder) is still open in the general case. We solve this problem for an even potential if the breather period equals twice the inverse velocity, and prove in that case the existence of exact travelling breather solutions superposed on an exponentially small periodic tail (in using results of [4]). A review of many results using the above method can be found in [3].

This is a joint work with G. James which generalizes the results of [2].

References:

[1] G. Iooss and K. Kirchgässner, *Travelling waves in a chain of coupled nonlinear oscillators*, Comm. Math. Phys. **211** (2000), 439–464.

[2] G. Iooss, *Travelling waves in the Fermi-Pasta-Ulam lattice*, Nonlinearity **13** (2000), 849–866.

[3] G. Iooss and G. James, *Localized waves in nonlinear oscillator chains*, Chaos **15** (2005), no. 1, 015113.

[4] G. Iooss and E. Lombardi, *Polynomial normal forms with exponentially small remainder for analytic vector fields*, J. Differential Equations **212** (2005), 1–61.