

**Joint AARMS-CRM Workshop:** Recent Advances in Functional  
and Delay Differential Equations  
1–5 November 2007

*Discontinuous Galerkin approximations for first  
kind Volterra integral equations*

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**Abstract**

We present new convergence results for piecewise polynomial discontinuous Galerkin (DG) approximations of a first kind Volterra integral equation of convolution kernel type, where the kernel  $K$  is smooth and satisfies  $K(0) > 0$ . We show that a  $m$ -th degree DG approximation exhibits global convergence of order  $m$  when  $m$  is odd and order  $m + 1$  when  $m$  is even. There is local superconvergence of one order higher (i.e. order  $m + 1$  when  $m$  is odd and  $m + 2$  when  $m$  is even), but in the even order case there is superconvergence only if the exact solution  $u$  of the equation satisfies  $u^{(m+1)}(0) = 0$ . We also present numerical test results which show that these theoretical convergence rates are optimal.

*This is joint work with Hermann Brunner and Dugald Duncan.*