Joint AARMS-CRM Workshop: Recent Advances in Functional and Delay Differential Equations
1–5 November 2007

Transition layers in a singularly perturbed differential- delay equation

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Abstract

A singularly perturbed differential-delay equation, which reduces to the logistic map as ε, the small parameter in the system is set to zero, is studied. In the period-doubling regime of the logistic map, this equation exhibits slowly oscillating periodic solutions of period approximately equal to 2. In particular, near the first period-doubling bifurcation of the logistic map, these periodic solutions resemble square waves for extremely small values of epsilon. This square-wave like solution consists of sharp transition layers of $O(\varepsilon)$ thickness connecting intervals of approximately unit length during each of which the solution has a nearly constant value close to one of the two period-2 fixed points of the map.

However if one measures two time intervals between three successive crossings of this solution with the midpoint of the two period-2 fixed points of the map, they are clearly different. Thus, the symmetry between the times spent in each of the two states, which is implicit in the continuous time logistic map is lost when epsilon is not zero. Using a two-parameter perturbative analysis, we solve the layer equations modelling the transitions and demostrate this subtle feature of these solutions. Our results are in excellent agreement with numerical calculations performed by a highly accurate spectral algorithm, which we will also present.