Progress on progressions

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We meet systems of homogeneous linear equations in high school algebra. They are especially easy to solve when there are more variables than equations. Infinitely many solutions! ... or so we are told. This talk aims to provide a counterpoint. Depending on what equation one is solving or where one is looking for solutions, one may find some, plenty, or none. We will explore the simplest geometric example of such an under-determined linear system, a k-term arithmetic progression, given by $(k-2)$ equations in k unknowns:

$$x_{i+1} - 2x_i + x_{i-1} = 0, \qquad 2 \leq i \leq k-1.$$

This example has a long and diverse history, with connections to several areas of analysis, number theory, combinatorics, and geometry. We will look at some of the landmark results in the subject and work out a couple of elementary proofs in real time.

This talk in intended for a general audience. No specialized knowledge will be assumed. Curiosity is a plus!