Why hyperbolic space disobeys social distancing

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The fundamental gap conjecture has been proved a few years ago after quite some time. It states that for any compact convex domain Ω in $\mathbb{R}^n$, with diameter D, the difference between the first two eigenvalues of the Dirichlet Laplacian on Ω (called the fundamental gap) satisfies the inequality $(\lambda_2 - \lambda_1)D^2 \geq 3\pi^2$. Soon afterwards, the exact same inequality was shown for convex domains on the sphere $\mathbb{S}^n$. Naturally, people inquired about the fundamental gap on the remaining space of constant curvature, the hyperbolic space $\mathbb{H}^n$.

In work with collaborators, T. Bourni, J. Clutterbuck, X.H. Nguyen, G. Wei and V.M. Wheeler, we have shown that for any diameter D, there exists a compact convex domain in $\mathbb{H}^n$ with diameter D for which the fundamental gap can be made arbitrarily small, hence ignoring distancing. I will outline the history of the problem, focusing on the similarities and differences between its features on Euclidean and hyperbolic spaces with the aim of giving an intuitive idea of why the fundamental gap conjecture fails in $\mathbb{H}^n$. In the process, I will also touch on different notions of convexity in the hyperbolic space.