CENTRE DE RECHERCHES MATHÉMATIQUES

Tenseurs : information quantique, complexité et combinatoires quantiques

Tensors: Quantum Information, Complexity and Combinatorics

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High-rank subtensors for high-rank tensors

A standard fact in linear algebra states that every matrix (over an arbitrary field) with rank k contains a k x k submatrix which also has rank k. A natural question is then whether this statement can be generalised to notions of rank on higher-dimensional tensors. In the case of the tensor rank, the optimistic generalisation is true: if d is a positive integer greater than or equal to 2 and if T is an order-d tensor with tensor rank equal to k, then T has a k x .. x k subtensor which still has tensor rank equal to k. The analogous statement for the slice rank is however false, as is shown by a counterexample constructed by Gowers using the Sawin-Tao method. We can nonetheless prove that for a class of notions R of rank on order-d tensors containing in particular the tensor rank, the slice rank, and the partition rank, the following asymptotic result holds. There exist functions F_{d,R}, G_{d,R} such that for every positive integer l, if T is an order-d tensor such that every subtensor T(X_1 x .. x X_d) of T obtained by restricting the entries of T to products of sets X_1,..,X_d of coordinates all with size at most F_{d,R}(l) has R-rank at most l, then T has R-rank at most G_{d,R}(l). In this talk we will discuss the proofs of some special cases, as well as some applications of the result and of the proof techniques.