CENTRE DE RECHERCHES MATHÉMATIQUES

Tenseurs : information quantique, complexité et combinatoires quantiques

Tensors: Quantum Information, Complexity and Combinatorics

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Polynomial-Time Power-Sum Decomposition of Polynomials

In this talk, I will present an efficient algorithm for finding power-sum decomposition of an input polynomial $P(x) = \sum_{i \le m} p_i(x)^d$ with component p_is. The case of linear p_is is equivalent to the well-studied tensor decomposition problem while the quadratic case occurs naturally in studying identifiability of non-spherical Gaussian mixtures from low-order moments.

Unlike tensor decomposition, both the unique identifiability and algorithms for this problem are not well-understood. For the simplest setting of quadratic p_is and $d = 3$, prior work of Ge, Huang and Kakade yields an algorithm only when $m \le \widetilde{O}(\sqrt{n})$. On the other hand, the more general recent result of Garg, Kayal and Saha builds an algebraic approach to handle any $m = n^{O(1)}$ components but only when d is large enough (while yielding no bounds for $d = 3$ or even $d = 100$) and only handles an inverse exponential noise.

Our results obtain a substantial quantitative improvement on both the prior works above even in the base case of $d = 3$ and quadratic p_is. Specifically, our algorithm succeeds in decomposing a sum of $m \sim \widetilde{O}(n)$ generic quadratic p_is for $d = 3$ and more generally the dth power-sum of $m \sim n^{2d/15}$ generic degree-K polynomials for any $K \ge 2$. Our algorithm relies only on basic numerical linear algebraic primitives, is exact (i.e., obtain arbitrarily tiny error up to numerical precision), and handles an inverse polynomial noise when the p_is have random Gaussian coefficients.

Based on joint work with Mitali Bafna, Pravesh K. Kothari and Jeff Xu (https://arxiv.org/pdf/2208.00122.pdf).