CENTRE DE RECHERCHES MATHÉMATIQUES

Tenseurs : information quantique, complexité et combinatoires quantiques

Tensors: Quantum Information, Complexity and Combinatorics

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An optimal inverse theorem for tensors over large fields

A tensor T(x_1, ..., x_n) is a multilinear function of the input vectors x_j in F_q^n, F_q a finite field. T has a small analytic rank if its output distribution is far from uniform. It has partition rank `r' if we can write T = f_1 * g_1 + ... + f_r * g_r, where f_r and g_r are tensors in fewer variables. Analytic rank measures the amount of randomness, and partition rank measures the amount of structure. It is known that if `f' has small partition rank, it must have small analytic rank. Green and Tao proved an inverse theorem stating that if `f' has small analytic rank then it has small partition rank. Their bound was qualitative, however, and several authors gave quantitative improvements. Janzer and Milicevic independently proved a polynomial dependence. We prove an optimal inverse theorem: the analytic rank and partition rank are equivalent up to constant factors over large enough fields. Our techniques are very different from the usual methods in this area, we rely on algebraic geometry rather than additive combinatorics. This is joint work with Guy Moshkovitz.