From Spherical DAHA to quantum Q-systems for classical root systems

Macondald and Koornwinder difference operators are elements in the functional representation of the spherical double affine Hecke algebras for root systems of type A or BC. The modular group acts on the spherical DAHA, and generates a distinguished set of operators which satisfy the quantized Q-systems for the classical root systems. These are creation operators for generalizations of q-Whittaker functions, which are graded characters of fusion products of KR-modules. In this talk I will explain the importance of the duality property for the universal solutions of the Macdonald or Koornwinder equations, and its relation to Toda theory, which provides the general proof of the integrability of the quantum Q-systems.