BOOK OF ABSTRACTS

SECTION 1 : PLENARY SPEAKERS

GÁBOR SZEGÖ PRIZE CONFERENCE
GENERALIZED LAMBERT SERIES

ATUL DIXIT

Abstract

Lambert series lie at the heart of modular forms and the theory of the Riemann zeta function. Among the early pioneers in the subject were Ramanujan and Wigert. We will discuss Ramanujan's formula for odd zeta values involving the Lambert series associated to n^k, where $k \neq -1$ is an odd integer, and its generalizations and analogues obtained by the speaker with his co-authors culminating into a recent transformation for $\sum_{n=1}^{\infty} \sigma_a(n) e^{-ny}$ for $a \in \mathbb{C}$ and $\mathrm{Re}(y) > 0$. We will also discuss several applications of this result. An analogue of a result of Wigert recently found by Soumyarup Banerjee, Shivajee Gupta and the author will be given along with its application in the zeta-function theory.

This talk is an amalagamation of results of the author on this topic from various papers co-authored with Bibekananda Maji, Rahul Kumar, Rajat Gupta, Soumyarup Banerjee, Shivajee Gupta and Aashita Kesarwani.

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THE STIELTJES–FEKETE PROBLEM AND DEGENERATE ORTHOGONAL POLYNOMIALS

TAMARA GRAVA

Abstract

A result of Stieltjes famously relates the zeroes of the classical orthogonal polynomials with the configurations of points on the line that minimize a suitable logarithmic energy, or equivalently the solutions of a suitable weighted Fekete problem. The optimal configuration satisfies an algebraic set of equations with the logarithmic derivative of the weight function as "external field": we call this set of algebraic equations the Stieltjes–Fekete problem. In this work we consider the Stieltjes-Fekete problem with an arbitrary rational external field. We show that its solutions are in one-to-one correspondence with the zeroes of certain non-hermitean orthogonal polynomials that satisfy an excess of orthogonality conditions and are thus termed "degenerate". This generalizes the above mentioned result of Stieltjes.

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SPECIAL FUNCTIONS IN QUANTUM STATISTICAL ESTIMATION

MASAHITO HAYASHI

Abstract

Estimating quantum system is one of crucial topics in quantum information. This talk focuses on estimating quantum dynamics, which is formulated as unitary estimation. In this problem, we have two freedoms to make a better estimation. One is the choice of the input quantum state, and the other is the choice of the quantum measurement. In this problem setting, we assume that the unknown unitary belongs to a subset of unitaries. For simplicity, we assume that the subset form a group, which simplifies the optimization of the choice of the measurement. Once our measurement is fixed in this way, the output distribution depends only on the choice of the input state. This part is formulated by Fourier transform even when the group composed of possible unitaries is a non-commutative group. In this problem, two types of special functions play central roles. One is prolate spheroidal wave function, which characterizes the minimum tail error probability of our esitmation in the one-parameteric setting. The other is the Mathieu funciton, which characterizes the minimum average error under the estimation of SU(2) action. This function is also used for the uncertainty relation on spheres $S^1 and S^3$.

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MOTION, MONODROMY AND Q-RIEMANN HILBERT PROBLEMS

NALINI JOSHI

Abstract

Newton was inspired by Kepler's laws of planetary motion to study motion on curves. This led him immediately to transcendental functions, that is, functions that cannot arise as solutions of polynomial equations. A century and a half later, the identification of new transcendental functions became a major topic in mathematics. I will give an overview of these results before considering monodromy: the study of how a function changes as its independent variable moves around a singularity. These two themes come together in the study of solutions of the Painlevé and discrete Painlevé equations. But there remain tantalizing open questions, especially for q-difference Painlevé equations. In this talk, we describe Riemann-Hilbert problems for q-difference equations and how to define monodromy surfaces explicitly for q-difference Painlevé equations.

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KOORNWINDER OPERATORS AND INTEGRABILITY OF QUANTUM Q-SYSTEMS

RINAT KEDEM

Abstract

We give a unified derivation of the q-difference realization of solutions of quantum Q-systems corresponding classical root systems, which are almost always quantum cluster algebras, and their conserved quantities. We use a specific collection of commuting Koornwinder/van Diejen operators. Upon specialization of the Koornwinder parameters, these correspond to various affine root systems, for which there exist known Q-systems. Their SL(2,Z)-translates, in the q-Whittaker limit, give the solutions of of the corresponding quantum Q-systems. By duality, we obtain corresponding generalizations of q-Toda Hamiltonians which are the conserved quantities for these systems. Their eigenfunctions are q-Whittaker functions. We explain the role of universal solutions to the Koornwinder equation and the Pieri equation in these constructions, and propose a duality property for these solutions.

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QUANTUM SYMMETRIC PAIRS AND BIVARIATE CONTINUOUS Q-HERMITE POLYNOMIALS

STEFAN KOLB

Abstract

Many constructions for quantum groups give rise to q-special functions. Quantum symmetric pairs were introduced in the 1990s to provide quantum group analogs of compact symmetric spaces which allow interpretations of Macdonald polynomials as zonal spherical functions. In the past 10 years, the theory of quantum symmetric pairs has been developing rapidly. In this talk I will review some of the recent developments. I will explain how a star-product interpretation of the underlying coideal subalgebras naturally gives rise to a new family of bivariate continuous q-Hermite polynomials and a family of deformed Chebyshev polynomials of the second kind. The talk is based on joint work with Riley Casper and Milen Yakimov.

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QUANTUM INVARIANT DIFFERENTIAL OPERATORS AND THEIR EIGENVALUES

GAIL LETZTER

Abstract

In this talk, we present the theory of quantum invariant differential operators from two perspectives: as central elements of the quantized enveloping algebra and as quantized Capelli operators inside quantum analogs of the Weyl algebra. We show that the center of the quantized enveloping algebra embeds inside the quantum Weyl algebra, thus enabling the transfer of results from the first setting to the second. Eigenvectors for central elements are identified with Macdonald polynomials and the eigenvalues are analyzed via a quantum Harish-Chandra map. Properties of the center obtained from this analysis are used to identify eigenvalues for the quantized Capelli operators with shifted Macdonald polynomials.

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CLASSICAL AND EXCEPTIONAL ORTHOGONAL POLYNOMIALS

ROBERT MILSON

Abstract

Exceptional Orthogonal Polynomials are orthogonal polynomial families that arise as solutions for second-order eigenvalue problems. They generalize the classical families of Hermite, Laguerre, and Jacobi in that they allow for polynomial sequences with a finite number of missing degrees. The fundamental technique for constructing such objects is the Darboux transformation, which ""dresses"" a classical operators to obtain orthogonal polynomials with a finite number of exceptional degrees.

Thanks to a foundational theorem that asserts that all exceptional orthogonal polynomials arise in precisely this fashion, it is now possible to envisage a complete classification of exceptional orthogonal operators and their attendant operators. In my talk I will describe the essential components of this programme and highlight the outstanding questions that must be answered en route to a full classification.

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HOLONOMIC FUNCTIONS AND MODULAR FORMS: AN ALGORITHMIC BRIDGE

PETER PAULE

Abstract

Holonomic functions and sequences satisfy linear differential and difference equations, respectively, with polynomial coefficients. It has been estimated that holonomic functions cover about 60 percent of the functions contained in the 1964 "Handbook" by Abramowitz and Stegun. A recent estimate says that holonomic sequences constitute about 20 percent of Sloane's OEIS database.

The study of these ubiquitous objects traces back to the time of Gauss (at least). Also tracing back to the time of Gauss (at least) are highly non-holonomic objects: modular functions and modular forms with q-series representations arising, for instance, as generating functions of partitions of various kinds.

Using computer algebra, the talk connects these two different worlds. Applications concern partition congruences, Fricke–Klein relations, irrationality proofs a la Beukers, or approximations to pi studied by Ramanujan and the Borweins. As a major ingredient to a "first guess, then prove" strategy, a new algorithm for proving differential equations for modular forms is used. The results presented arose in joint work with Silviu Radu (RISC).

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REVIVALS AND SPECIAL FUNCTIONS

BEATRICE PELLONI

Abstract

I will give an introduction to the phenomenon of "dispersive quantisation", or "revivals", an intriguing phenomenon that first observed and reported in 1835 by Talbot, after observing the pattern of light diffracted by a periodic grid. This phenomenon was only studied mathematically in the '90's, in particular for the periodic free space Schroedinger equation, by Berry and al - who called it the "Talbot effect". It was then rediscovered for the Airy equation by Peter Olver in 2010. Since then, a sizeable literature, summarised in the 2016 book of Erdogan & Tzirakis, has examined revivals for the periodic problem for linear dispersive equations with polynomial dispersion relation, as well as for the nonlinear Schrodinger and KdV equations. I will discuss a general definition of the revival property, in both a strong and weak form, and prove that it holds for more general boundary conditions. I will also illustrate a novel form of revivals for more general dispersion relations and nonlocal equations such as the linearised Benjamin-Ono equation, where the phenomenon can be understood in terms of a specific class of special functions . Finally, I will present results for nonlinear generalisations. This work is joint with Lyonell Boulton, George Farmakis, Peter Olver and David A. Smith.

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ON THE KANADE-RUSSELL IDENTITIES

HJALMAR ROSENGREN

Abstract

The Rogers-Ramanujan identities have intriguing relations to partition theory, orthogonal polynomials and affine Lie algebras. Using the relation to affine Lie algebras, Kanade and Russell conjectured many new Rogers-Ramanujan-type identities for triple series. Several of these conjectures were settled by Bringmann, Jennings-Shaffer and Mahlburg. We describe an approach to the Kanade-Russell identities, which leads both to new proofs of previously known identities and to conjectures that were still open. The new cases are based on quadratic transformations for Askey-Wilson polynomials.

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PLETHYSM AND THE ALGEBRA OF UNIFORM BLOCK PERMUTATIONS

ANNE SCHILLING

Abstract

We study the representation theory of the uniform block permutation algebra in the context of the representation theory of factorizable inverse monoids. The uniform block permutation algebra is a subalgebra of the partition algebra and is also known as the party algebra. We compute its characters and provide a Frobenius characteristic map to symmetric functions. This reveals connections of the characters of the uniform block permutation algebra and plethysms of Schur functions. This is based on joint work with Rosa Orellana, Franco Saliola and Mike Zabrocki.

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ORTHOGONAL POLYNOMIALS & QUANTUM WALKS: SCHUR CONNECTIONS

LUIS VELÁZQUEZ

Abstract

Since its very beginning, the theory of orthogonal polynomials (OP) has served as a meeting point for numerous pure and applied fields. This has resulted in a multiple symbiosis in which typical tools from OP theory have played a key role as communicating vessels. This talk intends to illustrate this fact, emphasizing its mathematical payoffs. The paradigm chosen for this starts with a recently found connection among several problems coming from apparently unrelated areas of mathematics and quantum theory. At the heart of these unexpected links is the notion of Schur function, one of the gems resulting from the interplay between harmonic analysis and complex variables, bequeathed to us by Issai Schur.

It turns out that very different mathematical and physical problems, formulated in the common language of Schur functions, feed into each other giving rise to a symbiotic spiral which ends in the solution to all of them. The talk will present these results, highlighting the essential role that the exchange of ideas among different fields may play in the solution of multiple problems.

The alluded results touch areas of OP theory, harmonic analysis, operator theory, random walks, as well as their quantum version, known as quantum walks.

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FROM CYCLOTOMIC POLYNOMIALS TO POLYNOMIALS ORTHOGONAL ON THE UNIT CIRCLE.

ALEXEI ZHEDANOV

Abstract

Cyclotomic polynomials are monic polynomials with zeros at primitive N-th roots of unity. Every cyclotomic polynomial generates two finite sequence of polynomials orthogonal on the unit circle (OPUC). The first sequence is similar to the Sturm sequence in the theory of polynomials orthogonal on the real line. The second sequence is related to Ramanujan's trigonometric sums. Many new explicit examples of OPUC are thus generated. Explicit expression of corresponding OPUC strongly depends on arithmetic properties of N.

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SECTION 2 : RICHARD ASKEY DAY SPEAKERS

DICK ASKEY IN INDIA

GEORGE ANDREWS

Abstract

This talk will focus on the 1987 collection of celebrations and conferences in India commemorating the 100th anniversary of Ramanujan's birth. Liz Askey kept an extensive diary that chronicles their entire tour of these events. I was with Dick and Liz at most of the happenings she describers. I will try to add colorful details form my own viewpoint and to give a sense of the importance of the role that Dick played in the continuing Ramanujan saga.

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HOW TO COMPUTE.....

JACOB CHRISTIANSEN

Abstract

In the talk, I'll tell about meeting Dick Askey (and his work) and how my visit to Madison in the fall of 2002 shaped me as a mathematician. Key words include Szego and graphs. And perhaps also teaching.

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WHAT I LEARNED FROM BEING A STUDENT OF ASKEY

SHAUN COOPER

Abstract

I will describe some of my experiences as a PhD student of Richard Askey, some of the topics he introduced me to and their ongoing influence on my career.

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ZEROS OF JACOBI POLYNOMIALS

KATHY DRIVER

Abstract

We discuss three problems posed by Richard Askey. The first involves a conjecture made by Dick in 1989 that the zeros of the equal degree Jacobi polynomials $P_n^{(\alpha,\beta)}(x)$ and $P_n^{(\alpha,\beta+2)}(x)$ are interlacing for each $\alpha, \beta > -1$. $n \in \mathbb{N}$. The second problem involves the identification of the sharp $t-$ interval for which the zeros of the Laguerre polynomials $L_n^{(\alpha)}(x)$ and $L_{n-1}^{(\alpha+t)}(x)$ are interlacing. The third problem arises from an asymptotic problem motivated by a result of Borwein and Chen which reduces to a connection between ultraspherical polynomials and ${}_2F_1$ hypergeometric polynomials. The three problems were solved in a more general form by D-Jordaan-Mbuyi; D-Muldoon and D-Duren respectively.

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58 YEARS OF SPECIAL FUNCTIONS AND HARMONIC ANALYSIS: ASKEY AS A NUCLEUS OF ACTIVITY

CHARLES F. DUNKL

Abstract

Part of this talk will be personal reminiscences in roughly chronological order. However the main idea is to present my overlook at Askey's influence, the people drawn into his aura, his emphasis on the underlying key ideas, and the resulting support of the research of younger mathematicians. The milestones are the meetings and conferences, notably Blacksburg 1974, Madison 1975, Bar-Le-Duc 1984. It was at these meetings that I met colleagues like George Gasper, Mourad Ismail, Tom Koornwinder and Willard Miller, each of whom shared their individual insights over the ensuing years, and encouraged me in productive research. No doubt Dick Askey was the force that brought us together. Note also that in the 60's and 70's there was no email or arXiv, the time between the writing of an article and the publication was generally more than a year, so that it was crucial to have contacts who were willing to share preprints. Thanks to Dick we established such a network. I will also remark on important special functions events since then up to the Askey 80 conference (Madison 2013).

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NOTES ON THE LAGRANGE INVERSION FORMULA

WARREN P. JOHNSON

Abstract

I will give several 18th and 19th century proofs of Lagrange's theorem for the expansion of a certain kind of implicit function in a power series, and perhaps a few applications. Dick Askey always encouraged this project, which I stopped working on some years ago, so this seems like an appropriate occasion to begin to revive it.

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DICK AND LIZ ASKEY'S WORLD TRIP IN FALL OF 1987

TOM H. KOORNWINDER

Abstract

During September 1987 - January 1988 Dick and Liz Askey made a long trip to U.S.S.R., Japan, Australia and India. On the way Dick gave many lectures and, in particular in India, attended some conferences as an invited speaker. But there was enough time left for making excursions, visiting museums and other cultural highlights, and getting acquainted with locals and with peculiarities of the various countries. Liz Askey wrote an extensive travel diary about this trip which was circulated among friends. From the diary we learn only superficially about Dick's mathematical activities, but we can read a lot about their cultural, historical and ethnographical explorations. The lecture will focus on the first three parts of their trip. The Indian part will be covered in the lecture by George Andrews, immediately after the present lecture. This lecture is in close collaboration with Suzanne Askey, daughter of Dick and Liz. The lecture is also meant as a tribute to Liz Askey, who passed away on January 29 of this year.

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THOSE WERE THE DAYS...

PAUL NEVAI

Abstract

TBA

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REMINISCENCES OF RICHARD ASKEY

DONALD RICHARDS

Abstract

I knew Dick Askey for over 36 years. In this talk, I will provide some reminiscences of Dick as mentor, co-author, and friend. My life was greatly improved by having met Dick, and I will always be grateful to him.

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THE ASKEY SCHEME AND ASKEY-WILSON POLYNOMIALS

DENNIS STANTON

Abstract

I will review motivation for the Askey scheme and hypergeometric polynomials leading to the Askey-Wilson polynomials. This is from the 1970's. I will also recall some ideas Dick told us that did not turn out as he expected. I will indicate how the same ideas can be used for recent work with Jang Soo Kim and Mourad Ismail on type R_I polynomials.

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SECTION 3 : MINISYMPOSIUM SPEAKERS

SECTION 3 : MINISYMPOSIUM SPEAKERS

#1 Minisymposium in combinatorics of special functions

NEW LINEAR RELATIONS AMONG LLT POLYNOMIALS

DONGHYUN KIM

Abstract

LLT polynomial is one of a family of symmetric functions introduced by Lascoux, Leclerc, and Thibon as q-analogues of products of Schur functions. In this talk we discuss new linear relations among LLT polynomials and as an application we describe a certain linear combination of Macdonald polynomials related to Science Fiction conjecture by Bergeron and Garsia.

Based on an ongoing joint work with Seung Jin Lee and Jaeseong Oh.

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RAISING MACDONALD POLYNOMIALS FROM YOUNG

JENNIFER MORSE

Abstract

Young raising operator formulas for classical families of symmetric functions are the natural starting point for developing the theory of symmetric functions. However, this perspective has been missing from the contemporary theory growing up around Macdonald polynomials since the 1980's.

We give a raising operator formula for Macdonald polynomials which straightforwardly recovers classical formulas for Hall-Littlewood and Schur functions. Our method reveals that Macdonald polynomials are part of a larger class of conjecturally Schur positive symmetric functions and brings forth a wealth of new combinatorial, representation theoretic and geometric questions.

This work is joint with Blasiak, Haiman, Pun, and Seelinger

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NEW DELTA AND THETA OPERATOR IDENTITIES IN THE THEORY OF MACDONALD POLYNOMIALS

ROMERO, MARINO

Abstract

In recent times, Macdonald polynomials and their related operators have had an increasing role in a number of different areas of study. To demonstrate a few of their important properties, we will start by giving a brief history of some of the connections between modified Macdonald polynomials, representation theory, and geometry. In particular, we will look at Delta eigenoperators and Theta operators, which have recently been conjectured to give the distributions of irreducible representations of the symmetric group in certain multigraded modules. We are going to present several new identities and conjectures regarding Delta and Theta operators, and we will demonstrate how a new class of objects we call γ-parking functions give a framework for understanding the combinatorics surrounding these operators. This talk is based on joint work with Alessandro Iraci.

References

[1] Alessandro Iraci and Marino Romero, *Delta and Theta Operator Expansions*, arXiv:2203.10342 (2020).

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COMPATIBILITY AND COMPANIONS FOR LEONARD PAIRS

PAUL TERWILLIGER

Abstract

In this talk, we introduce the notions of compatibility and companion for Leonard pairs. These concepts are roughly described as follows. A Leonard pair is an ordered pair A, A^* of diagonalizable linear maps on a finite dimensional vector space V, that each act in an irreducible tridiagonal fashion on an eigenbasis for the other one. Leonard pairs A, A^* and B, B^* on V are called compatible whenever $A^* = B^*$ and $[A, A^*] = [B, B^*]$, where $[r, s] = rs - sr$. For a Leonard pair A, A^* on V, a companion of A, A^* is a linear map $K : V \to V$ such that K is a polynomial in A^* and $A - K, A^*$ is a Leonard pair on V. We will explain how compatibility and companion are related. In our main results, we find all the Leonard pairs B, B^* that are compatible with a given Leonard pair A, A^*. For each solution B, B^* we describe the corresponding companion. This is joint work with Kazumasa Nomura.

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LINEAR RELATIONS AMONG SCHUBERT STRUCTURE COEFFICIENTS

ANNA WEIGANDT

Abstract

Schubert structure coefficients describe the multiplicative structure of the cohomology rings of flag varieties. Much work has been done on the problem of giving combinatorial formulas for these coefficients in special cases, as well as on the related problem of identifying vanishing and nonvanishing conditions. We establish families of linear relations among Schubert structure coefficients, which allow one to discern properties of unknown coefficients from properties of others. Building on work of Wyser, we give new formulas for certain Schubert structure coefficients in terms of (p,q)-clans. This is joint work with Oliver Pechenik.

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SYMMETRIC GROUP CHARACTER BASIS OF SYMMETRIC FUNCTIONS AND COMBINATORIAL REPRESENTATION THEORY

MIKE ZABROCKI

Abstract

I will define an orthonormal basis of the symmetric functions which are the characters of the irreducible representations of the symmetric groups in the same way that Schur functions are the characters of the irreducible representations of the general linear group. I will discuss how these symmetric functions are related to several open problems in combinatorial representation theory. This is joint work with Laura Colmenarejo, Rosa Orellana, Franco Saliola and Anne Schilling.

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SECTION 3 : MINISYMPOSIUM SPEAKERS

#2 Application of orthogonal polynomials and special functions to quantum information.

PROGRAMMING QUANTUM GATES

GIULIO CHIRIBELLA

Abstract

A universal quantum processor is a device that can approximately implement arbitrary quantum gates, whose description is encoded in a suitable program. In most implementations of quantum computing, the program consists of classical data. From the foundational point of view, however, it is interesting to explore the more general scenario where the program is itself a quantum system. In the past two decades, a major open question has been to determine how the size of the smallest quantum program scales with the required accuracy in the implementation of the desired gate. Here we answer the question, by proving a bound on the size of the program and designing a concrete protocol that attains the bound in the asymptotic limit. Our result is based the representation theory of the special unitary group. It provides improved bounds on the estimation of unitary gates, and on the implementation of quantum protocols subject to conservation laws.

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TENSORS: RANK, ENTROPY AND ENTANGLEMENT

MATTHIAS CHRISTANDL

Abstract

We wish to understand when a tensor s can be transformed into a tensor t by application of linear maps to its tensor legs (we then say s restricts to t). In the language of restrictions, the rank of a tensor t is given by the minimal size of a diagonal tensor restricting to t. The study of rank and restrictions are motivated by algebraic complexity theory, where the rank corresponds to the computational complexity of a bilinear map (e.g. matrix multiplication) which then is viewed as a tensor with three legs.

Interestingly, some important open problems can be formulated in terms of asymptotic properties of restriction, among them the exponent of matrix multiplication. Following the seminal work of Volker Strassen, we will therefore study whether for large n the (n+o(n))'th tensor power of s can be restricted to the n'th tensor power of t. The information-theoretic flavor of the problem is apparent and was heavily used by Strassen in conjunction with the discovery of algebraic structures (his spectral theorem).

Identifying k-leg-tensors with states of quantum systems of k particles allows us to bring tools and ideas from quantum information theory to the table, among them entanglement polytopes and quantum entropy. I will use these to construct a family of functionals - the quantum functionals - that can serve as obstructions to asymptotic restrictions. The functionals are the first of their kind applicable to all tensors, thereby solving a problem by Strassen from 1986.

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LEARNING AND TESTING QUANTUM STATES

JOHN WRIGHT

Abstract

In the area of quantum state learning, one is given a small number of ""samples"" of a quantum state, and the goal is use them to determine a feature of the state. Examples include learning the entire state (""quantum state tomography""), determining whether it equals a target state (""quantum state certification""), or estimating its von Neumann entropy. These are problems which are not only of theoretical interest, but are also commonly used in current-day implementation and verification of quantum technologies.

In this talk, I will describe recent work giving efficient algorithms for a variety of these problems, including the first optimal algorithms for tomography and state certification. These results make use of a new connection between quantum state learning and longest increasing subsequences of random words, a topic in combinatorics dating back to a 1935 paper of Erdős and Szekeres. Finally, I will show how this connection motivates new and interesting questions about the length of the longest increasing subsequence in random words, some of which we have found optimal answers for.

Based on joint works with Costin Badescu and Ryan O'Donnell.

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NONEXISTENCE OF S-DISTANCE 2S-DESIGNS ON PROJECTIVE UNITARY GROUPS

ZIQING XIANG

Abstract

A SIC-POVM is 1-distance 2-design on $C^d/U(1)$. The s-distance 2s-designs on protective unitary groups PU(d) = U(d) / U(1) can be viewed as higher strength higher dimensional analog of SIC-POVM. We determine the size of an s-distance 2s-design on projective unitary groups using two methods: (1) using Fisher type bound (2) using associated orthogonal polynomials. These two sizes are different when s is at least 2, which implies that there are no s-distance 2s-designs on projective unitary groups. The proof involves orthogonal polynomials and their super analogs, some generalizations of Hook-length formula, and some interesting Hankel determinant calculations.

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A DISCRETE PROBABILITY DISTRIBUTION EXPRESSED BY RACAH POLYNOMIAL FROM SCHUR-WEYL DUALITY

SHINTAROU YANAGIDA

Abstract

This talk is based on the collaboration (arXiv:2104.12635) with Masahito Hayashi (SUSTech/Nagoya Univ.) and Akihito Hora (Hokkaido Univ.). Considering the irreducible decomposition of a certain distinguished vector in the tensor space of the classical SU(2)-Sn Schur-Weyl duality, we obtained a discrete probability distribution equipped with four parameters. Surprisingly, this distribution can be expressed by Racah polynomial, which sits in the top line of Askey scheme of hypergeometric orthogonal polynomials. Moreover, the cumulative distribution function is expressed by a 4F3-hypergeometric polynomial. I would like to give an overview of this distribution, including the asymptotic analysis and the limit continuous distribution.

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REPRESENTATION MATCHING FOR REMOTE QUANTUM COMPUTING

YUXIANG YANG

Abstract

Many quantum computational tasks have inherent symmetries, suggesting a path to enhancing their efficiency and performance. Exploiting this observation, we propose representation matching, a generic probabilistic protocol for reducing the cost of quantum computation in a quantum network. We show that the representation-matching protocol is capable of reducing the communication or memory cost to almost the minimum in various tasks, including remote execution of unitary gate arrays, permutation gates, and unitary conjugation, as well as the storage and retrieval of unitary gates.

Paper reference: arXiv 2009.06667/PRX Quantum 2, 020327

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CLIFFORD GROUP AND T-DESIGNS

HUANGJUN ZHU

Abstract

A unitary t-design is a set of unitaries that is "evenly distributed" in the sense that the average of any 2t-th order polynomial over the design equals the average over the entire unitary group. It is a generalization of complex projective designs on the complex unit sphere and spherical designs on the real unit sphere. In various fields – e.g. quantum information theory – one frequently encounters constructions that rely on matrices drawn uniformly at random from the unitary group. Often, it suffices to sample these matrices from a unitary t-design, for sufficiently large t. This results in more explicit, derandomized constructions. The most prominent unitary t-design considered in quantum information is the multi-qubit Clifford group. It is known to be a unitary 3-design, but, unfortunately, not a 4-design. Here, we first discuss the structure of the Clifford group and its connections with t-designs. Then we discuss various applications of the Clifford group and t-designs in quantum information processing.

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SECTION 3 : MINISYMPOSIUM SPEAKERS

#3 All things hypergeometric (classical, basic and elliptic) and q-series generalisations

PARTITION IDENTITIES FOR k-REGULAR PARTITIONS WITH DISTINCT PARTS

GEORGE E. ANDREWS

Abstract

We start with a little-known Euler type theorem (due to Alladi) which is the following: The number of partitions of n into distinct parts not divisible by k (i.e., k-regular partitions with distinct parts) equals the number of partitions of n into odd parts none repeated more than $k-1$ times. $k=1$ and 2 are tautologies. $k=3$ plays a prominent role in Schur's 1926 partition theorem. Both Alladi and Schur have further partition identities related to $k=2$ which we will discuss. Obviously, $k=\infty$ is Euler's theorem. We then proceed to $k=4$ where an empirical investigation leads to a result for overpartitions. We conclude with a proof of the $k=4$ case and look at results and possibilities for $k>4$.

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TELESCOPING CONTINUED FRACTIONS FOR THE ERROR TERM IN STIRLING'S FORMULA

GAURAV BHATNAGAR

Abstract

We introduce telescoping continued fractions to find lower bounds for the error term r_n in Stirling's approximation $n! = \sqrt{2\pi} n^{n+1/2} e^{-n} e^{r_n}$. This improves lower bounds given earlier by Cesàro (1922), Robbins (1955), Nanjundiah (1959), Maria (1965) and Popov (2017). The expression is in terms of a continued fraction, together with an algorithm to find successive terms of this continued fraction. The technique we introduce allows us to experimentally obtain upper and lower bounds for a sequence of convergents of a continued fraction in terms of a difference of two continued fractions.

Joint work with Krishnan Rajkumar.

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FINE'S FUNCTION AND PARTIAL THETA FUNCTION

HENG HUAT CHAN

Abstract

In this talk, we discuss a generalization of an identity due to G.E. Andrews and S.O. Warnaar. The identity of Andrews and Warnaar was used to simplify a proof of a generalization of Jacobi's Triple Product Identity. Our generalization of their identity (using the work of N.J. Fine) turns out to be easy to remember if we know the original identity.

Joint work with Song Heng Chan, Kuo-Jye Chen and Sen Shan Huang.

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THE ANTISYMMETRY RELATION FOR CONTINUOUS q-JACOBI POLYNOMIALS

HOWARD S. COHL

Abstract

By comparing Gauss hypergeometric representations of the antisymmetric Jacobi polynomials and the Gegenbauer function of the first kind one may obtain a proportional relation between these two objects. The Gegenbauer function of the first kind is the standard continuation of the Gegenbauer (ultraspherical) polynomials when the degree of the polynomial is a complex number. In order to obtain a q-analogue of this relation, we use Rahmans AskeyWilson function of the first kind which is the continuation of the AskeyWilson polynomial when its degree is allowed to be complex. We specialize the Rahmans AskeyWilson function of the first kind to obtain two types of continuous q-Jacobi functions of the first kind and corresponding continuous q-ultraspherical/Rogers functions of the first kind. Using these functions we derive q-analogues of the antisymmetry relation for continuous q-Jacobi polynomials.

Joint work with Roberto S. Costas-Santos.

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HYPERGEOMETRIC FUNCTIONS, DIFFERENCE EQUATIONS AND WRONSKIANS

ROBERTO S. COSTAS-SANTOS

Abstract

There are several ways to prove that a real number x is irrational. Among them, perhaps the most popular is related to the fact that if a_n/b_n are rational approximations of x, then both a_n and b_n satisfy the same difference equation. In this talk we will see how it is common for the solutions of this type of difference equations to be hypergeometric functions, and how to apply certain Wronskians to the sequences a_n, b_n, $\ldots$, in order to find suitable rational approximations related to the real number x.

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FURTHER q-SUPERCONGRUENCES FROM A TRANSFORMATION OF RAHMAN

VICTOR J. W. GUO

Abstract

Employing a quadratic transformation formula of Rahman and the method of 'creative microscoping' (introduced by the author and Zudilin in 2019), we provide some new q-supercongruences for truncated basic hypergeometric series. In particular, we confirm two recent conjectures of Liu and Wang. We also propose some related conjectures on supercongruences and q-supercongruences.

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DUALITY RELATIONS FOR HYPERGEOMETRIC AND BASIC HYPERGEOMETRIC FUNCTIONS

DMITRY KARP

Abstract

Reduction formulas for the sum of products of hypergeometric functions can be traced back to Euler. This topic has intimate relations with summation and transformation formulas, contiguous relations and algebraic properties of the (generalized) hypergeometric differential equation. Over the last decade a number of important discoveries have been made in this subject by various authors and using algebraic, combinatorial and analytic techniques. In the talk we will discuss recent progress both for the ordinary and the basic hypergeometric functions based on analytic techniques. Our results generalize a number of known identities due to Gorelov, Ebisu, Kuznetsov, Feng and Young, Suzuki and, partially, Beukers and Jouhet.

Joint work with Alexey Kuznetsov (York University, Canada) and Sergey Kalmykov (Shanghai Jiao Tong University, China).

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TRIANGULAR RECURRENCES AND HYPERGEOMETRIC TRANSFORMATIONS

ROBERT S. MAIER

Abstract

Many triangles of numbers arise in combinatorics and the theory of special functions. The best known are Pascal's triangle, and the triangular arrays that contain Stirling numbers or the generalized Stirling numbers of Hsu and Shiue, Lah numbers, Bessel numbers, Narayana numbers, and Eulerian numbers. These all belong to the class of GKP (Graham–Knuth–Patashnik) triangles, generated by triangular recurrences of the GKP type. An approach to solving such recurrences, and transforming such recurrences and the associated row polynomials, began with Wilf. He noted that the corresponding bivariate exponential generating functions (EGF's) can be expressed in terms of the Gauss hypergeometric function. We show that the group of transformations of GKP recurrences or their EGF's, recently applied by Salas and Sokal, can be viewed as the group of permutations of the three singular points of the Gauss hypergeometric ODE. This leads to the definition of a new class of generalized Eulerian numbers, related to the generalized Stirling numbers. Moreover, there is a class of transformations of GKP EGF's which is analogous to the quadratic transformations of the Gauss hypergeometric function. The existence of such quadratic transformations explains the recently observed connection between Narayana polynomials and Eulerian polynomials.

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A METHOD FOR SUMMING q-BESSEL SERIES

ZEINAB MANSOUR

Abstract

For $\mu, \nu > -1$, we consider the q-Bessel series

$$U_{\mu,\nu}(x;q^2) = \frac{(1-q^2)^\mu}{(qx)^\mu} \sum_{m=1}^{\infty} \frac{a_m}{j_{m,\nu}^\mu} J_\mu^{(3)}(qj_{m,\nu}x;q^2),$$

where $(j_{m,\nu})$ are the positive zeros of the third Jackson q-Bessel function $J_\nu^{(3)}(\cdot;q^2)$ and (a_m) is a sequence of real numbers satisfying $\sum_m |a_m|/(J_{m,\nu}^{\mu+1/2}) < \infty$. We propose a q-analog of the method introduced in [Proc. Amer. Math. Soc. 150 (2022), 763-778] for computing in a closed form the sum of the Bessel series $U_{\mu,\nu}$ assuming that for a particular value η of the parameter μ a closed expression for $U_{\eta,\nu}$ as a power series in x is known. Examples are included. Joint work with Karema Oraby

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ON ANOTHER CHARACTERIZATION OF THE ASKEY–WILSON POLYNOMIALS

DIEUDONNÉ MBOUNA

Abstract

In this talk we expose the theory of classical orthogonal polynomials on lattices and we use this to give a Al-Salam and Chihara type characterization of classical orthogonal polynomials on lattices.

Joint work with A. Suzuki.

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SOME MORE IDENTITIES OF KANADE-RUSSELL TYPE

JAMES MC LAUGHLIN

Abstract

In this talk we consider some variations and generalizations of the multi-sum to single-sum transformation recently used by Rosengren in his proof of the Kanade-Russell identities.

These general transformations are then used to prove a number of identities equating multi-sums and infinite products or multi-sums and infinite product $\times$ a false theta series. Examples include the following:

$$\sum_{j,k,p,r=0}^{\infty} \frac{(-1)^{j+k} q^{(2j+k-p+r)^2/2+k(k+4)/2+3j-p/2+3r/2} (-q;q)_r}{(q^2;q^2)_j (q;q)_k (q;q)_p (q;q)_r} = 2\frac{(-q;q^2)_\infty (-q^2,-q^{14},q^{16};q^{16})_\infty}{(q;q)_\infty}.$$

Let

$$Q(i,j,k,l,p) := \frac{1}{2}(i+6j+4k+2l-p)(i+6j+4k+2l-p-1) + 2k(k-1)+l(l-1)+3i+15j+14k+5l-2p.$$

Then

$$\sum_{i,j,k,l,p=0}^{\infty} \frac{(-1)^{l+k} q^{Q(i,j,k,l,p)}}{(q;q)_i (q^6;q^6)_j (q^4;q^4)_k (q^2;q^2)_l (q;q)_p} = \frac{2(-q;q)_\infty^2}{q(q^3;q^6)_\infty (q^4;q^4)_\infty}\left(1+\sum_{r=1}^{\infty}\left(q^{9r^2+6r}-q^{9r^2-6r}\right)\right),$$

$$\sum_{j,k,p=0}^{\infty} (-1)^k \frac{q^{(3j+2k-p)(3j+2k-p-1)/2+k(k-1)-p+6j+6k}}{(q^3;q^3)_j (q^2;q^2)_k (q;q)_p} = \frac{(-1;q)_\infty (q^{18};q^{18})_\infty}{(q^3;q^3)_\infty (q^9;q^{18})_\infty}.$$

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LARGE-DEGREE ASYMPTOTIC EXPANSIONS FOR THE LEGENDRE AND ALLIED FUNCTIONS

GERGŐ NEMES

Abstract

I shall discuss the asymptotic behaviour of the Legendre-, Ferrers- and Gegenbauer functions when their degree becomes large. I will present factorial- and inverse factorial expansions of these functions complete with explicit, computable error bounds.

Joint work with Adri B. Olde Daalhuis.

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NEW IDENTITIES FOR MEIJER'S G-FUNCTION

ELENA PRILEPKINA

Abstract

The talk is devoted to the study of the piecewise-analytical case of Meijer G-function. Similarly to the Miller–Paris transformations for the generalized hypergeometric functions we present identities for the Meijer G-function having integer parameter difference. Next, we give some curious integral involving G-function and its corollaries in the form of summation formulas. Particular attention will be paid to the analytic continuation of the G-function from the interior to the exterior of the unit disk. Simple formulas for such a continuation will be presented. We also study the behavior of the principal on the banks of the branch cut.

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A DISCRETE AND q-ASYMPTOTIC ITERATION METHOD

NASSER SAAD

Abstract

We introduce a finite difference and q-difference analogues of the Asymptotic Iteration Method of Ciftci, Hall, and Saad. We give necessary, and sufficient condition for the existence of a polynomial solution to a general linear second-order difference or q-difference equation subject to a "terminating condition", which is precisely defined. When a difference or q-difference equation has a polynomial solution, we show how to find the second solution.

Joint work with Mourad E. H. Ismail.

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AN ELLIPTIC CHAUNDY–BULLARD IDENTITY

MICHAEL SCHLOSSER

Abstract

An identity mentioned by Chaundy and Bullard in 1960 (and often attributed to these two authors in the literature) expresses $1/(1-x)^n$ $(n = 1, 2, \ldots)$ as a sum of two truncated binomial series. This identity has been reproved and rediscovered many times in different areas of mathematics; nevertheless, it has actually already occurred in famous treatises by de Montmort (1713) and de Moivre (1738) on probability theory. We discuss various extensions of the Chaundy–Bullard identity, with an emphasis on q-analogues and generalizations containing extra parameters. In particular, in 2009, Xinrong Ma, in the quest of establishing variants of Ramanujan's reciprocity theorem, found a q-analogue of the Chaundy–Bullard identity containing three free parameters. We present a q-analogue with four free parameters that can be viewed as an identity for balanced and very-well-poised basic hypergeometric series and that admits a direct elliptic extension.

Joint work with Natsuko Hoshi, Makoto Katori, and Tom Koornwinder.

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BEST ALGEBRAIC BOUNDS FOR RATIOS OF MODIFIED BESSEL FUNCTIONS

JAVIER SEGURA

Abstract

The best possible algebraic bounds of the form $B(\alpha,\beta,\gamma,x)=(\alpha+\sqrt{\beta^2+\gamma^2x^2})/x$ for ratios of modified Bessel functions (both of the first and second kinds) are characterized. The bounds are sharp as $x\to 0^+$ and/or as $x\to+\infty$ and at any given $x_*>0$, and they are the best possible bounds around such x_* (a different bound for each value of x_*). Bounds with maximal accuracy at 0^+ and $+\infty$ are recovered in the limits $x_*\to 0^+$ and $x_*\to+\infty$, and for these cases the coefficients have simple expressions. For the case of finite and positive x_* we provide uniparametric families of bounds which are close to the optimal bounds and retain their confluence properties.

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DISCRETE ORTHOGONALITY OF THE ASKEY–WILSON POLYNOMIALS AND OTHER FAMILIES IN THE q-ASKEY SCHEME

LUIS VERDE-STAR

Abstract

For some families of basic hypergeometric orthogonal polynomials, that include the Askey–Wilson family, we construct a discrete weight function w defined on a sequence of distinct nodes x_k such that

$$\tau(p(t)) = \sum_{k=0}^{\infty} p(x_k)w(x_k),$$

for any polynomial $p(t)$, where τ is the quasi-definite moment functional that corresponds to an orthogonal polynomial sequence $u_n(t)$ in the q-Askey scheme. The nodes $x_k = b_0 + b_1 q^k + b_2 q^{-k}$, with at least one of b_1 and b_2 nonzero, determine the sequence $v_k(t)$ of Newton polynomials. Let $m_k = \tau(v_k(t))$ be the generalized moments of τ with respect to the Newton sequence. We find the weights $w(x_k)$ by solving the system of equations

$$m_k = \tau(v_k(t)) = \sum_{j=0}^{\infty} v_k(x_j)w(x_j),$$

where $k \geq 0$, using the inverse of the infinite matrix $[v_k(x_j)]$. For the most general family in the q-Askey scheme the weights are of the form

$$w(x_j) = K_j \; {}_3\phi_2(q^j y_1, q^j y_2, q^j y_3; q^j r, q^{2j} s; q; q),$$

where y_1, y_2, r, s are parameters, $y_3 = rs/qy_1y_2$ and

$$K_j = (-1)^j q^{j(j-1)/2} \frac{(y_1;q)_j (y_2;q)_j (y_3;q)_j}{(q;q)_j (r;q)_j (q^{j-1}s;q)_j}.$$

For the Askey–Wilson polynomials with parameters a, b, c, d we obtain $y_1 = ab, \quad y_2 = ac, \quad y_3 = ad, \quad r = abcd, \quad s = qa^2.$

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THE HEUN EIGENVALUE PROBLEM AND SOME OF ITS SPECIAL CASES

HANS VOLKMER

Abstract

The standard eigenvalue problem for the Heun differential equation is studied. In particular, its numerical treatment is addressed. Several eigenvalue problems for the Lamé equation are shown to be special cases of the Heun eigenvalue problem. These Lamé eigenvalue problems arise in the solution of the Laplace equation in three dimensional Euclidian space by the method of separation of variables in cyclide coordinate systems. These are the flat-ring, bicyclide and flat-disk co-ordinates.

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CYLINDRIC PARTITIONS AND THE ANDREWS–GORDON IDENTITIES FOR $\mathrm{A}_2^{(1)}$

S. OLE WARNAAR

Abstract

The Rogers–Ramanujan and Andrews–Gordon identities are some of the most well-studied q-series identities in all of mathematics. In this talk I will report on a generalisation of the Rogers–Ramanujan and Andrews–Gordon identities to the affine Lie algebra $\mathrm{A}_2^{(1)}$. Key ingredients in our work are cylindric partitions and a new A_2 Bailey lemma.

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ORTHOGONAL POLYNOMIALS OF ASKEY-WILSON TYPE

RUIMING ZHANG

Abstract

We study two families of orthogonal polynomials. The first is a finite family related to the Askey–Wilson polynomials but the orthogonality is on $\mathbb{R}$. A limiting case of this family is an infinite system of orthogonal polynomials whose moment problem is indeterminate. We provide several orthogonality measures for the infinite family and derive their Plancherel–Rotach asymptotics.

Joint work with Mourad E. H. Ismail and Keru Zhou.

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SECTION 3 : MINISYMPOSIUM SPEAKERS

#4 Orthogonal polynomials in Sobolev spaces and related topics

A CLASS OF SOBOLEV ORTHOGONAL POLYNOMIALS ON THE UNIT CIRCLE AND ASSOCIATED CONTINUOUS DUAL HAHN POLYNOMIALS

CLEONICE BRACCIALI

Abstract

We present orthogonal polynomials and associated connection coefficients with respect to a class of Sobolev inner products on the unit circle. Under certain conditions on the parameters in the inner product it is shown that the connection coefficients are related to a subfamily of the continuous dual Hahn polynomials. Properties regarding bounds and asymptotics are also established with respect to these parameters. Criteria for knowing when the zeros of the (Sobolev) orthogonal polynomials and also the zeros of their derivatives stay within the unit disk have also been addressed. Joint work with Jéssica V. da Silva, and A. Sri Ranga.

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SEMICLASSICAL ORTHOGONAL POLYNOMIALS AND NONLINEAR DIFFERENTIAL EQUATIONS

GALINA FILIPUK

Abstract

In this talk I shall explain how the study of nonlinear differential equations can shed some light on the theory of semiclassical orthogonal polynomials and vice versa. This is a joint work with A. Stokes and A. Dzhamay. We present a connection between systems of differential equations for the recurrence coefficients of polynomials orthogonal with respect to the generalized Meixner and the deformed Laguerre weights. It is well-known that the recurrence coefficients of both generalized Meixner and deformed Laguerre orthogonal polynomials can be expressed in terms of solutions of the fifth Painleve equation but no explicit relation between systems of differential equations for the recurrence coefficients was known. We also present certain limits in which the recurrence coefficients can be expressed in terms of solutions of the Painleve XXXIV equation, which in the deformed Laguerre case extends previous studies and in the generalized Meixner case is a new result.

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COHERENT PAIRS AND SOBOLEV-TYPE ORTHOGONAL POLYNOMIALS ON THE REAL LINE: AN EXTENSION TO THE MATRIX CASE

LUIS E. GARZA GAONA

Abstract

In this contribution, we extend the concept of coherent pair for two quasi-definite matrix linear functionals u_0 and u_1. Necessary and sufficient conditions for these functionals to constitute a coherent pair are determined, when one of them satisfies a matrix Pearson-type equation. Joint work with Edinson Fuentes.

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BISPECTRAL JACOBI TYPE POLYNOMIALS

MANUEL DOMÍNGUEZ DE LA IGLESIA

Abstract

We study the bispectrality of Jacobi type polynomials, which are eigenfunctions of higher-order differential operators and can be defined by taking suitable linear combinations of a fixed number of consecutive Jacobi polynomials. Jacobi type polynomials include, as particular cases, the Krall-Jacobi polynomials. As the main results we prove that the Jacobi type polynomials always satisfy higher-order recurrence relations (i.e., they are bispectral). We also prove that the Krall-Jacobi families are the only Jacobi type polynomials which are orthogonal with respect to a measure on the real line. Joint work with A. J. Durán.

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THE ONE-DIMENSIONAL KREIN LAPLACIAN SELF-ADJOINT OPERATOR AND SEQUENCES OF KREIN-SOBOLEV ORTHOGONAL POLYNOMIALS

LANCE L. LITTLEJOHN

Abstract

This is joint work with Alejandro Quintero Roba (Baylor).

The one-dimensional Krein Laplacian operator A in $L^2[a,b]$ is the self-adjoint operator A given by $Af = -f'' + kf$ for $f \in \mathcal{D}(A)$, where

$$\mathcal{D}(A) = \left\{ f : [a,b] \to \mathbb{C} \left| \begin{matrix} f, f' \in AC[a,b] \\ f, f'' \in L^2[a,b] \end{matrix} \right. ; \begin{pmatrix} f(b) \\ f'(b) \end{pmatrix} = \begin{pmatrix} 1 & b-a \\ 0 & 1 \end{pmatrix} \begin{pmatrix} f(a) \\ f'(a) \end{pmatrix} \right\}.$$

Here we take $k > 0$. It is straightforward to see that A is bounded below in $L^2[a,b]$ by kI; that is to say

$$(Af, f)_{L^2[a,b]} \geq k(f,f)_{L^2[a,b]}.$$

Consequently, from a general left-definite operator theory developed by Littlejohn and Wellman, there is a continuum of left-definite Hilbert-Sobolev spaces $\{(H_r, (\cdot,\cdot)_r)\}_{r>0}$ associated with $(L^2[a,b], A)$. For example, when $r = 1$, the first left-definite space is given by

$$H_1 = \{f : [a,b] \to \mathbb{C} \mid f \in AC[a,b]; f' \in L^2[a,b]\}$$

and the first left-definite inner product on H_1 is given by

$$(f,g)_1 = -\frac{(f(b)-f(a))(\overline{g}(b)-\overline{g}(a))}{b-a} + \int_a^b (f'(x)\overline{g}'(x) + kf(x)\overline{g}(x))\, dx.$$

For $r = 2$, $H_2 = \mathcal{D}(A)$ and

$$\begin{aligned} (f,g)_2 &= -2k\frac{(f(b)-f(a))(\overline{g}(b)-\overline{g}(a))}{b-a} \\ &\quad + \int_a^b \left(f''(x)\overline{g}''(x) + 2kf'(x)\overline{g}'(x) + k^2 f(x)\overline{g}(x)\right) dx. \end{aligned}$$

For each $r \in \mathbb{N}$, the set of all polynomials is dense in H_r; we construct the corresponding orthogonal polynomials in each H_r.

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A SECOND–ORDER DIFFERENCE/DIFFERENTIAL EQUATION FOR GENERAL SOBOLEV TYPE ORTHOGONAL POLYNOMIALS

JUAN F. MAÑAS-MAÑAS

Abstract

We consider a general Sobolev–type inner product involving the Hahn difference operator, so this includes the well–known difference operators $\mathbb{D}_q$ and Δ and, as a limit case, the derivative operator. The objective is to construct the ladder operators for the corresponding nonstandard orthogonal polynomials and in this way we deduce the second–order difference equation satisfied by these polynomials. Moreover, we will show that all the functions involved in these constructions can be computed explicitly. This is a joint work with Galina Filipuk and Juan J. Moreno–Balcázar.

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ON SOBOLEV ORTHOGONALITY OF POLYNOMIAL SOLUTIONS OF SECOND ORDER PARTIAL DIFFERENTIAL EQUATIONS

MISAEL E. MARRIAGA

Abstract

Given a second order linear partial differential operator L with polynomial coefficients of degree at most 2, we study the orthogonality of its polynomial eigenfunctions with respect to a Sobolev bilinear form (.,.). This bilinear form is constructed with moment functionals, and satisfies the symmetry condition (L[p],q)=(p,L[q]) for all polynomials p and q. Moreover, the involved moment functionals must satisfy related Pearson equations. We use Koornwinder's method to construct solutions to these Pearson equations and to construct a complete polynomial system of eigenfunctions of L.

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SOBOLEV ORTHOGONALITY ON THE UNIT BALL

MIGUEL PIÑAR

Abstract

"We study the orthogonal structure on the d-dimensional unit ball with respect to the Sobolev inner product involving the gradients and a mass point at the origin.

Extending the results obtained by Y. Xu, an explicit family of Sobolev orthogonal polynomials on the unit ball is constructed. This family is given in terms of the classical orthogonal polynomials on the unit ball. As a consequence, we show that, for a given function f, the Fourier-Sobolev coefficients can be computed without involving the partial derivatives of f. Finally, some estimates are given for the error of best approximation by polynomials in the Sobolev space.

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GENERALIZED MIXED TYPE BERNOULLI-GEGENBAUER AND LAGRANGE-BASED HYPERGEOMETRIC BERNOULLI POLYNOMIALS

YAMILET QUINTANA

Abstract

The aim of this talk is to present a brief report about two new families of special polynomials generated via the generating function method. We will explore some of the algebraic and analytic properties for these families, including, a matrix-inversion formula for each one of them. Part of this talk is based on joint work with Azhar Iqbal and Waseem A. Khan, Prince Mohammad Bin Fahd University, and Sahar Albosaily, University of Ha'il.

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COHERENT PROPERTY OF THE SECOND KIND AND SOBOLEV ORTHOGONAL POLYNOMIALS

ALAGACONE SRI RANGA

Abstract

Given two positive measures, or more generally, two semidefinite linear functionals, we obtain necessary and sufficient conditions such that the pair satisfies a property which we call coherence property of the second kind. For example, we show that in this case both measures are semiclassical of class s=1. Many explicite examples of such pairs of moment functional are found. In the case the moment functionals are positive definite, Sobolev orthogonal polynomials associated with these pairs of moment functionals (in these cases represented by positive measures) are also studied.

This is a joint work with Francisco Marcellán (UC3M), Gustavo A. Marcato (UNESP) and Mijael H. Suni (UNESP).

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ON THE SECOND-ORDER HOLONOMIC EQUATION FOR SOBOLEV-TYPE ORTHOGONAL POLYNOMIALS

MARIA DAS NEVES REBOCHO

Abstract

In this talk it is considered a general approach to the study of orthogonal polynomials related to Sobolev inner products which are defined in terms of divided-difference operators having the fundamental property of leaving a polynomial of degree n-1 when applied to a polynomial of degree n. The main focus is on analytic properties for the orthogonal polynomials, including the second-order holonomic difference equation satisfied by them.

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SOME PARTIAL DIFFERENTIAL EQUATIONS FOR SOBOLEV TYPE ORTHOGONAL POLYNOMIALS ON SEVERAL VARIABLES

HERBERT DUEÑAS RUIZ

Abstract

We present partial differential equations that satisfy some particular examples of high order Sobolev-Type orthogonal polynomials of several variables. Examples on product domains, unit ball, the simplex and on the cone are presented. The results on [1], [2] and [3] are used.

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RECURRENCE RELATIONS AND SOME OTHER PROPERTIES OF SOBOLEV ORTHOGONAL POLYNOMIALS

MURAD SULTANAKHMEDOV

Abstract

We consider the specific Sobolev-type inner products, recently in number of works by Sharapudinov I.I., which were shown to be a very useful apparatus for different applied tasks including simultaneous approximation of functions and their derivatives, approximate solution of differential and difference equations, etc. We managed to obtain recurrence relations for polynomials orthonormal with respect to Sobolev-type inner products of this kind and generated by classical orthogonal polynomials, namely: Jacobi polynomials, Legendre polynomials, Chebyshev polynomials of the first and the second kind, Gegenbauer (ultraspherical) polynomials, Hermite polynomials. We also establish similar formulas for Sobolev orthogonal polynomials and generated by classical orthogonal polynomials of discrete variable, namely: discrete Chebyshev polynomials, Meixner polynomials, Charlier polynomials and Kravchuk polynomials. Finally, we will consider some other important and useful properties of Sobolev orthogonal polynomials in more details.

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OPERATOR PENCILS AND SOBOLEV ORTHOGONAL POLYNOMIALS

SERGEY ZAGORODNYUK

Abstract

By operator pencils one means polynomials with operator coefficients, acting in a Banach space. The simplest case is a linear pencil of the form $A - xB$, which is related to the following eigenvalue problem: $Ap = xBp$. It turned out that pencils of difference and differential operators appeared in a natural way in the domain of Sobolev orthogonal polynomials. In particular, it appeared that a lot of linear differential equations with polynomial coefficients of higher orders have Sobolev orthogonal polynomials as their generalized eigenfunctions. On the other hand, we shall discuss the following difference pencils: (1) $J_{2N+1} - xE$, which leads to orthogonal polynomials on rays with a discrete Sobolev term, as well as to the matrix orthogonal polynomials; (2) $J_5 - xJ_3$, which yields special orthogonality relations, and some examples of Sobolev orthogonal polynomials. Here J_n are symmetric semi-infinite banded matrices with n non-zero diagonals.

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SECTION 3 : MINISYMPOSIUM SPEAKERS

#5 Integrable Systems, Orthogonal Polynomials and Special Functions

THE PADÉ PROBLEM ON RIEMANN SURFACES AND MATRIX ORTHOGONAL POLYNOMIALS

MARCO BERTOLA

Abstract

The main goal of the talk is to connect matrix polynomial biorthogonality on a contour in the plane with a suitable notion of scalar, multi-point Padé approximation on an arbitrary Riemann surface endowed with a rational map to the Riemann sphere. To this end we introduce an appropriate notion of (scalar) multi-point Padé approximation on a Riemann surface and corresponding notion of biorthogonality of sections of the semi-canonical bundle (half-differentials). Several examples are offered in illustration of the new notions.

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THE COMPLEX ELLIPTIC GINIBRE ENSEMBLE AT WEAK NON-HERMITICITY

THOMAS BOTHNER

Abstract

In this talk we will focus on the complex elliptic Ginibre ensemble (eGinUE) and analyze the statistical behavior of its eigenvalues in a suitable scaling limit, known as the weak non-Hermiticity limit. In this limit the asymmetry parameter in the model scales with the matrix dimension and the so obtained 2D limiting point processes generalize the well-known sine and Airy processes from the Gaussian unitary ensemble. Using integro-differential Painlevé transcendents we will show how the gap functions of the 2D limiting point processes can be evaluated in closed form and how Riemann-Hilbert techniques can subsequently yield precise asymptotic information for the same functions. Based on current joint work with Alex Little.

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A RIEMANN-HILBERT APPROACH FOR ALGEBRAIC SOLUTIONS OF THE PAINLEVE-III (D7) EQUATION

ROBERT BUCKINGHAM

Abstract

The D7 degeneration of the Painleve-III equation has algebraic solutions for certain parameter values. Using the isomonodromy method, we build a Riemann-Hilbert representation of these solutions. In the large-parameter limit we rigorously determine the asymptotic behavior in the singularity-free region and compute the boundary of the singularity region.

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CLASSICAL SOLUTIONS OF THE FIFTH PAINLEVÉ EQUATION

PETER CLARKSON

Abstract

In this talk I will discuss classical solutions of the fifth Painlevé equation ($\mathrm{P_V}$)

$$\frac{\mathrm{d}^2 w}{\mathrm{d}z^2} = \left(\frac{1}{2w} + \frac{1}{w-1}\right)\left(\frac{\mathrm{d}w}{\mathrm{d}z}\right)^2 - \frac{1}{z}\frac{\mathrm{d}w}{\mathrm{d}z} + \frac{(w-1)^2(\alpha w^2+\beta)}{z^2 w} + \frac{\gamma w}{z} + \frac{\delta w(w+1)}{w-1}, \tag{1}$$

where α, β, γ and δ are constants.

The general solutions of the Painlevé equations are transcendental in the sense that they cannot be expressed in terms of known elementary functions. However, it is well known that all Painlevé equations except the first equation possess rational solutions, algebraic solutions and solutions expressed in terms of the classical special functions for special values of the parameters. These solutions of the Painlevé equations are often called "classical solutions" and frequently can be expressed in the form of determinants.

In the generic case of $\mathrm{P_V}$ (1) when $\delta \neq 0$, special function solutions are expressed in terms of Kummer functions and has rational solutions expressed in terms of Laguerre polynomials. In the case of $\mathrm{P_V}$ (1) when $\delta = 0$, which is known as deg-$\mathrm{P_V}$ and related to the third Painlevé equation, special function solutions are expressed in terms of Bessel functions and has algebraic solutions expressed in terms of Laguerre polynomials. I shall give some new representations of some of these classical solutions and discuss some applications.

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ASYMPTOTICS OF THE DEFORMED FREDHOLM DETERMINANT OF THE CONFLUENT HYPERGEOMETRIC KERNEL

DAN DAI

Abstract

In this paper, we consider the deformed Fredholm determinant of the confluent hypergeometric kernel. This determinant represents the gap probability of the corresponding determinant point process where each particle is removed independently with probability $1-\gamma$, $0 \leq \gamma < 1$. We derive asymptotics of the deformed Fredholm determinant when the gap interval tends to infinity, up to and including the constant term. As an application of our results, we establish a central limit theorem for the eigenvalue counting function and a global rigidity upper bound for its maximum deviation.

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SPECIAL FUNCTION SOLUTIONS OF PAINLEVÉ DIFFERENTIAL EQUATIONS

ALFREDO DEAÑO

Abstract

In this talk we present an overview of recent results regarding special function solutions of the Painlevé differential equations. These solutions appear in connection with orthogonal polynomials on the real line and in the complex plane, as well as with random matrix theory and numerical analysis. The asymptotic analysis of such solutions can be carried out in different regimes and using both classical methods, such as multivariate steepest descent, and the Riemann-Hilbert formulation. This is based on joint work with A. Barhoumi (U. of Michigan), A. Celsus (U. of Cambridge), D. Huybrechs (KU Leuven), A. Iserles (U. of Cambridge) and D. Ruiz-Antolín (U. de Cantabria).

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ON RATIONAL SOLUTIONS OF PAINLEVE EQUATIONS

CLARE DUNNING

Abstract

We explore some intriguing combinatorial aspects of certain polynomials that form rational solutions of the Painleve equations and their higher-order analogues.

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RECURRENCE RELATIONS FOR THE GENERALIZED LAGUERRE AND CHARLIER ORTHOGONAL POLYNOMIALS AND DISCRETE PAINLEVÉ EQUATIONS ON THE D-6 SAKAI SURFACE

ANTON DZHAMAY

Abstract

In this talk we consider two examples of certain recurrence relations, or nonlinear discrete dynamical systems, that appear in the theory of orthogonal polynomials, from the point of view of Sakai's geometric theory of Painlevé equations. On one hand, this gives us new examples of the appearance of discrete Painlevé equations in the theory of orthogonal polynomials. On the other hand, it serves as a good illustration of the effectiveness of a recently proposed procedure on how to reduce such recurrences to some canonical discrete Painlevé equations. Of particular interest is the fact that both recurrences are regularized on the same family of rational algebraic surfaces, but at the same time their dynamics are non-equivalent. This is a joint work with Galina Filipuk (University of Warsaw) and Xing Li and Da-jun Zhang (Shanghai University).

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NONLINEAR DIFFERENTIAL EQUATIONS AND THE GEOMETRIC APPROACH

GALINA FILIPUK

Abstract

Nonlinear differential equations may have complicated singularities in the complex plane. Painleve equations are nonlinear second order differential equations solutions of which have no movable critical points. They have a lot of nice properties. The quasi-Painleve equations admit algebraic branch points. The geometric approach to the Painleve equations was developed in the works of K. Okamoto, H. Sakai and many others. In this talk I shall present some recent results using the geometric approach applied to second order nonlinear differential equations.

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LADDER OPERATORS AND DIFFERENTIAL EQUATIONS FOR MATRIX ORTHOGONAL TIME-DEPENDENT JACOBI POLYNOMIALS

ASSIL FRADI

Abstract

Under some integrability conditions We find explicit formulas for the left and right raising and lowering firstorder differential operators for orthogonal matrix "time-dependent Jacobi" polynomials. We derivere-currence relations for the left and right coefficients in the raising and lowering operators. We also derive asecond-order differential equation satisfied by these polynomials. we show how to use the compatibilityconditions to explicitly determine the recurrence coefficients of the monic Jacobi polynomials. The non-Abelian extensions of a family of discrete Painlevé IV equations are discussed.

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COMPLETE CLASSIFICATION OF RATIONAL SOLUTIONS OF A_2N-PAINLEVÉ SYSTEMS

DAVID GÓMEZ-ULLATE

Abstract

We provide a complete classification and an explicit representation of rational solutions to the fourth Painlevé equation and its higher order generalizations known as the A2n-Painlevé or Noumi-Yamada systems. The construction of solutions makes use of the theory of cyclic dressing chains of Schrödinger operators. Studying the local expansions of the solutions around their singularities we find that some coefficients in their Laurent expansion must vanish, which express precisely the conditions of trivial monodromy of the associated potentials. The characterization of trivial monodromy potentials with quadratic growth implies that all rational solutions can be expressed as Wronskian determinants of suitably chosen sequences of Hermite polynomials. The main classification result states that every rational solution to the A_2n-Painlevé system corresponds to a cycle of Maya diagrams, which can be indexed by an oddly coloured integer sequence. Finally, we establish the link with the standard approach to building rational solutions, based on applying Bäcklund transformations on seed solutions, by providing a representation for the symmetry group action on coloured sequences and Maya cycles. The special polynomials associated with higher order A_2n-Painlevé systems generalize the Okamoto and generalized Hermite polynomials which appear in the rational solutions of PIV (n=1).

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MATRIX VALUED ORTHOGONAL POLYNOMIALS AND RANDOM TILING MODELS

ARNO KUIJLAARS

Abstract

I will discuss how matrix valued orthogonal polynomials appear in the analysis of random tiling models with periodic weightings. The matrix valued orthogonality can be viewed as scalar orthogonality on a Riemann surface.

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THE MAXWELL-BLOCH SYSTEM IN THE SHARP-LINE LIMIT

PETER MILLER

Abstract

We study the characteristic Cauchy problem for the Maxwell-Bloch system that describes the interaction of an optical pulse with an active quantum medium. It is well known that in the sharp-line limit that the atoms in the medium are not Doppler-shifted in frequency, this system can be embedded in the integrable hierarchy of the nonselfadjoint Zakharov-Shabat spectral problem. However, it is also known that there are certain difficulties with formulating and using the inverse-scattering transform based on this spectral problem in the usual way. We construct a Riemann-Hilbert problem that returns the unique causal solution of the Cauchy problem and use it to explain features of solutions such as the stimulated decay by a suitable optical pulse of an unstable medium to its stable state and the spontaneous generation of a dispersive tail of the optical pulse with positive time that ruins absolute integrability that would be needed for the standard inverse-scattering transform to make sense. This tail is related to a specific self-similar solution of the Maxwell-Bloch system that in turn is connected with a concrete special solution of the Painlevé-III equation that has become important in several recent application problems for the focusing nonlinear Schrödinger equation. This is joint work with Sitai Li (Xiamen).

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SUPERINTEGRABILITY, PAINLEVÉ TRANSCENDENTS AND EXCEPTIONAL ORTHOGONAL POLYNOMIALS

SARAH POST

Abstract

In this talk, I will briefly review connections between exceptional orthogonal polynomials (EOPs) and Painlevé equation as illuminated by their connection with superintegrable systems. I will give an overview of different places where these connections arise and focus on recent work with Ian Marquette and Lisa Ritter connecting exceptional Jacobi polynomials with Painlevé VI.

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ON Q-PAINLEVÉ VI, SINGULAR SEGRE SURFACES AND ORTHOGONAL POLYNOMIALS

PIETER ROFFELSEN

Abstract

In this talk, I will discuss a correspondence between solutions of q-Painlevé VI and points on an explicit Segre surface, obtained in recent work with Nalini Joshi. Upon fixing a point on the latter surface, one can in principle compute the values of the corresponding solution by solving an associated Riemann-Hilbert problem. For special values of the parameters, the correspondence ceases to be one-to-one and the surface develops singularities. I will show how the Riemann-Hilbert problem simplifies drastically at such singularities and can be solved explicitly in terms of certain orthogonal polynomials.

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GAP PROBABILITIES FOR THE BURES-HALL ENSEMBLE AND DEFORMED CAUCHY-LAGUERRE BI-ORTHOGONAL POLYNOMIALS.

NICHOLAS, WITTE

Abstract

The Bures metric and the associated Bures-Hall measure is arguably the natural choice for studying the spectrum of the quantum mechanical density matrix with no apriori knowledge about a quantum system. This current understanding arose in response to the question - What is a typical quantum state? Employing this model we investigate the probability of a gap in the spectrum of the density matrix, either at the bottom $[0, s)$ or at the top $(s, 1]$, utilising the connection of this Pfaffian point-process with the allied problem in the determinantal point-process of a deformed Cauchy-Laguerre bi-orthogonal polynomial system. To this end we develop new general results about Cauchy bi-orthogonal polynomial systems for general classes of weights: in particular a new, simple Christoffel-Darboux formula, reproducing kernels and differential equations for the polynomials and their associated functions. This two-dimensional system is most simply expressed with rank-3 matrix variables and possesses an associated bilinear form. Furthermore under specialisation to Laguerre type densities in the weight we construct differential equations in two deformation variables s, t, and observe that the recurrence, spectral and deformation derivative structures form a compatible and integrable triplet of Lax equations. An explicit system of constrained dynamical equations in s, t governs this system.

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SECTION 3 : MINISYMPOSIUM SPEAKERS

#6 Multiple orthogonal polynomials and Hermite-Padé approximation

Q - DIFFERENCE EQUATIONS OF HIGH ORDER AND WKB ASYMPTOTICS FOR COLORED JONES POLYNOMIALS

ALEXANDER I. APTEKAREV

Abstract

We consider q-difference equations for colored Jones polynomials. These polynomials are invariants for the knots and their asymptotics plays an important role in the famous Volume Conjecture (VC) for the complement of the knot to the 3d sphere. We state a Theorem on asymptotic expansion of general solutions of the q-difference equations. For the partial solutions, corresponding to the colored Jones polynomials, using some heuristic and numeric consideration, we suggest a conjecture on their WKB asymptotics. For the special knots under consideration this conjecture is in accordance with VC.

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SOME EXPANSION FORMULAS FOR BRENKE POLYNOMIAL SETS.

HAMZA CHAGGARA

Abstract

In this paper, we derive some expansion formulas associated to Brenke polynomials. The obtained results will be applied to the generalized Gould-Hopper polynomials.

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SPECTRAL THEORY OF JACOBI MATRICES ON TREES WHOSE COEFFICIENTS ARE GENERATED BY MULTIPLE ORTHOGONAL POLYNOMIALS

SERGEY DENISOV

Abstract

The polynomials orthogonal on the real line (OPRL) give rise to Jacobi matrices and many problems in spectral theory can be solved using this connection and sharp results known for OPRL. In this talk, I will explain the relation between multiple orthogonal polynomials and Jacobi matrices on the rooted trees. The basics of the spectral theory and some applications will be reviewed. Based on joint work with Maxim Yattselev.

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GLOBAL FLUCTUATIONS FOR MULTIPLE ORTHOGONAL POLYNOMIAL ENSEMBLES VIA NEAREST NEIGHBOR RECURRENCES

MAURICE DUITS

Abstract

This talk will report on joint work B. Fahs and R. Kozhan on the fluctuations of linear statistics with polynomial test functions for Multiple Orthogonal Polynomial Ensembles. Multiple Orthogonal Polynomial Ensembles form an important class of determinantal point processes that include random matrix models such as the GUE with external source, complex Wishart matrices, multi-matrix models and others. Our analysis is based on the recurrence matrix for the multiple orthogonal polynomials, that is constructed out of the nearest neighbor recurrences. If the coefficients for the nearest neighbor recurrences have limits, then we show that the right-limit of this recurrence matrix is a matrix that can be viewed as representation of a Toeplitz operator with respect to a non-standard basis. This will allow us to prove Central Limit Theorems for linear statistics for Multiple Orthogonal Polynomial Ensembles.

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ELECTROSTATIC PARTNERS AND ZEROS OF ORTHOGONAL AND MULTIPLE ORTHOGONAL POLYNOMIALS

ANDREI MARTINEZ-FINKELSTEIN

Abstract

The well-known electrostatic interpretation of the zeros of Hermite, Laguerre or Jacobi polynomials, which goes back to the 1885 work of Stieltjes, is one of the most popular models in the theory of orthogonal polynomials. It was picked up and extended to several contexts, such as orthogonal and quasi-orthogonal polynomials on the real line and the unit circle, for classical and semiclassical weights. Our first goal is to generalize the known electrostatic interpretations. For a given polynomial P with simple zeros, and a given semiclassical weight W, we present a construction that yields a linear second-order differential equation (ODE), and in consequence, an electrostatic model for zeros of P. The coefficients of this ODE are written in terms of a dual polynomial that we call the electrostatic partner of P. This construction is absolutely general and can be carried out for any polynomial with simple zeros and any semiclassical weight on the complex plane. An additional assumption of quasi-orthogonality of P with respect to W allows us to give more precise bounds on the degree of the electrostatic partner. In the case of orthogonal and quasi-orthogonal polynomials, we recover some of the known results and generalize others. For the Hermite-Padé or multiple orthogonal polynomials of type II, this approach yields a system of linear second-order differential equations, from which we derive an electrostatic interpretation of their zeros in terms of a vector equilibrium. More detailed results are obtained in the special cases of Angelesco, Nikishin, and generalized Nikishin systems. We also discuss the discrete-to-continuous transition of these models in the asymptotic regime, as the number of zeros tends to infinity, into the known vector equilibrium problems. This is a joint work with R. Orive (Universidad de La Laguna, Canary Islands, Spain) and J. Sanchez-Lara (Granada University, Spain).

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RANDOM WALKS VERSUS MULTIPLE ORTHOGONAL POLYNOMIALS

ANA FOULQUIÉ

Abstract

The Karlin-Mc Gregor representation formula links the theory of orthogonal polynomials and stochastic processes. If we describe Markov chains beyond birth and deaths chains, we can extend this Karlin-Mac Gregor representation formula, now in terms of multiple orthogonal polynomials. We have explore this connection giving explicit examples of transition matrices and obtaining an spectral theorem for a class of bounded oscillatory Hessenberg matrices that represent these Markov chains. This is a joint work with Amílcar Branquinho and Manuel Mañas.

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EIGENVALUE FLUCTUATIONS OF MULTIPLE ORTHOGONAL POLYNOMIAL ENSEMBLES

ROSTYSLAV KOZHAN

Abstract

We show that global fluctuations of linear eigenvalue statistics of Multiple Orthogonal Ensemble of random matrices is asymptotically normal for polynomial test functions and compute its limiting variance. The class of measures that we allow is quite general: we only require existence (after, potentially, a rescaling) of the nearest neighbour recurrence coefficients along rays. In particular, this includes multiple Hermite, multiple Laguerre, multiple Charlier, and mutliple Krawtchouk ensembles. This is a joint work with B. Fahs and M. Duits.

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STRONG ASYMPTOTIC OF MULTI-LEVEL HERMITE-PADÉ POLYNOMIALS

GUILLERMO LOPEZ LAGOMASINO

Abstract

We find the strong asymptotic of multiple orthogonal polynomials arising in a mixed type Hermite-Padé interpolation scheme with respect to a Nikishin system generated by a collection of measures which verify Szegő's condition. The result is a natural extension of Szegő's classical theorem to multiple orthogonality. An interesting feature is that the proof uses Banach's and Brouwer's fixed point theorems.

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MULTIPLE ORTHOGONAL POLYNOMIALS AND BRANCHED CONTINUED FRACTIONS

HELDER LIMA

Abstract

The central theme of this talk is the recently found connection between two completely different corners of mathematics: multiple orthogonal polynomials and branched continued fractions introduced to solve total-positivity problems arising from combinatorics. Firstly, we give an overview of the connection between these two topics. Then, we give further evidence of this connection via the case study of the link between branched-continued-fraction representations for ratios of contiguous hypergeometric series and multiple orthogonal polynomials with respect to measures (or linear functionals) whose moments are ratios of products of Pochhammer symbols. Specialisations of these multiple orthogonal polynomials include the classical Laguerre, Jacobi, and Bessel orthogonal polynomials, multiple orthogonal polynomials with respect to Nikishin systems of two measures involving modified Bessel functions, confluent hypergeometric functions, and Gauss' hypergeometric function, and multiple orthogonal polynomials with respect to Meijer G-functions used to investigate the singular values of products of Ginibre random matrices. This is a collaboration with Alan Sokal (UCL).

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JACOBI MATRICES ON BINARY TREES: BOUNDEDNESS AND MULTILEVEL INTERPOLATIONS

VLADIMIR LYSOV

We consider tridiagonal Jacobi matrices (or so-called discrete Schrödinger operators) on graphs. On homogeneous trees one of the methods to implement such operators is based on the Hermite–Padé interpolation problems for perfect systems, see [1].

Analogous to the classic case, the Jacobi matrices are determined by the recurrence coefficients for orthogonal polynomials. For one important class of perfect systems, the so-called Nikishin systems, it is known [2] that such a construction leads to unbounded operators.

We show that another interpolation problem can be posed for the Nikishin systems. So its solutions also satisfy the recurrent relations, and the corresponding coefficients and the Jacobi operator become bounded, see [3].

This is a joint work with Alexander Aptekarev (Keldysh Institute of Applied Mathematics).

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ON DIFFERENTIAL RELATIONS OF 2-ORTHOGONAL POLYNOMIALS AND HAHN'S PROPERTY

TERESA A. MESQUITA

Abstract

Considering the differential operator on the vectorial space of polynomial functions presented in [1], a recent new approach [2] allowed us to obtain explicitly polynomial eigenfunctions. At the same time, we prove some results about the classical character of solutions regarding specific differential operators. In this talk, we present the main results on these topics, adding new aspects to the global study of 2-orthogonal polynomial solutions of differential identities defined by means of a third order differential operator that does not increase the degree of polynomials.

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APPLICATION OF MULTIPLE ORTHOGONAL POLYNOMIALS FOR CONSTRUCTION OF THE OPTIMAL SET OF QUADRATURE RULES IN THE SENSE OF BORGES

TATJANA V. TOMOVIĆ MLADENOVIĆ

Abstract

We consider the problem of numerically evaluating a set of $r \in \mathbb{N}$, $r \geq 2$, definite integrals of the form

$$\int_E f(x)\, w_j(x) \mathrm{d}x, \quad j = 1, 2, \ldots, r,$$

where w_j, $j = 1, 2, \ldots, r$, are the weight functions, i.e., optimal set of quadrature rules in the sense of Borges for numerically evaluating a set of $r \in \mathbb{N}$, $r \geq 2$, definite integrals with the same integrand and over the same interval of integration, but with different weight functions. By using multiple orthogonal polynomials the stable numerical method for the construction of such quadrature rules for two and three weight functions was given in [1] and for four weight functions in [2].

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LATTICE PATHS, VECTOR CONTINUED FRACTIONS, AND RESOLVENTS OF BANDED HESSENBERG OPERATORS

ABEY LÓPEZ-GARCÍA AND VASILIY PROKHOROV

Abstract

We give a combinatorial interpretation of vector continued fractions obtained by applying the Jacobi-Perron algorithm to a vector of $p \geq 1$ resolvent functions of a banded Hessenberg operator of order $p + 1$.

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STRONG ASYMPTOTIC OF CAUCHY BIORTHOGONAL POLYNOMIALS

LUIS GIRALDO GONZÁLEZ RICARDO

Abstract

We give the strong asymptotic of Cauchy biorthogonal polynomials under the assumption that the defining measures are supported on bounded non intersecting intervals of the real line and satisfy Szegö's condition. The biorthogonal polynomials are connected with certain mixed type Hermite-Padé polynomials, which verify full orthogonality relations with respect to certain varying measures. Thus, the strong asymptotic of orthogonal polynomials with respect to varying measures plays a key role in the study.

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ON A CONJECTURE BY MILOVANOVIC̀ AND DORDEVIC̀ ABOUT ZEROS OF HUMBERT POLYNOMIALS

NEILA BEN ROMDHANE

Abstract

Using some new results on zeros of d-orthogonal polynomials, we solve a conjecture by Milovanovic̀ and Dordevic̀ about Humbert polynomials for most values of the given parameters.

In this talk, we explain how algebraic curves and equilibrium problems can be considered on equal footing. We consider a family of MOPs with polynomial potentials, and we associate each MOP to an algebraic equation. The algebraic equation depends on the degree of the MOP at hand. Under a natural boundedness condition on associated recurrence coefficients, in the large degree limit the algebraic equation admits a limiting curve which, in turn, describes the limiting zero distribution of the MOPs.

With the limiting curve at hand, we move further and show that they can be used to construct vector critical measures, which are saddle points of an energy functional. One of the vector components is supported on an S-contour on the plane, whose main features are also described in detail.

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LIMITING ZERO DISTRIBUTION OF MOPS AND THEIR ALGEBRAIC CURVES AND EQUILIBRIUM PROBLEMS

GUILHERME SILVA

Abstract

Zeros of orthogonal polynomials of large degree for polynomial potentials are classically described either by equilibrium problems or algebraic curves, and these two different characterizations are, in a sense, equivalent. When moving to multiple orthogonality, however, equilibrium problems have been playing the major role in the characterization of the limiting behavior of zeros, with algebraic curves being solely a support character in the play.

In this talk, we explain how algebraic curves and equilibrium problems can be considered on equal footing. We consider a family of MOPs with polynomial potentials, and we associate each MOP to an algebraic equation. The algebraic equation depends on the degree of the MOP at hand. Under a natural boundedness condition on associated recurrence coefficients, in the large degree limit the algebraic equation admits a limiting curve which, in turn, describes the limiting zero distribution of the MOPs.

With the limiting curve at hand, we move further and show that they can be used to construct vector critical measures, which are saddle points of an energy functional. One of the vector components is supported on an S-contour on the plane, whose main features are also described in detail.

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MULTIPLE ORTHOGONAL POLYNOMIALS AND BRANCHED CONTINUED FRACTIONS

ALAN SOKAL

Abstract

I analyze an unexpected connection between multiple orthogonal polynomials, d-orthogonal polynomials, production matrices and branched continued fractions. This work can be viewed as a partial extension of Viennot's combinatorial theory of orthogonal polynomials to the case where the production matrix is lower-Hessenberg but is not necessarily tridiagonal.

[This talk is for the mini-symposium on "Multiple orthogonal polynomials and Hermite-Padé approximation".]

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MULTIPLE ORTHOGONAL POLYNOMIALS ON THE SEMICIRCLE

MARIJA STANIC

Abstract

In this talk two types of multiple orthogonal polynomials on the semicircle with respect to a set of r different weight functions are considered. Such polynomials are generalizations of polynomials orthogonal on the semicircle with respect to a complex–valued inner product $[f,g] = \int_0^\pi f\left(\mathrm{e}^{\mathrm{i}\theta}\right) g\left(\mathrm{e}^{\mathrm{i}\theta}\right) w\left(\mathrm{e}^{\mathrm{i}\theta}\right) \mathrm{d}\theta$. We present proof of the existence and uniqueness of such multiple orthogonal polynomials for certain classes of weight functions, as well as some their properties. Also, an application in numerical integration is given.

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ON A NEW APPROACH TO CONSTRUCTIVE APPROXIMATION OF MULTIVALUED ANALYTIC FUNCTIONS

SERGEY SUETIN

Abstract

We will discuss some new approach to constructive approximation of a multivalued analytic function f. The corresponding rational functions are based on type I Hermite-Pade polynomials for tuple of functions $[1, f, \ldots, f^m]$.

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CHRISTOFFEL FUNCTIONS FOR MULTIPLE ORTHOGONAL POLYNOMIALS

GRZEGORZ ŚWIDERSKI

Abstract

We study weak convergence of the Christoffel-Darboux kernel on the diagonal corresponding to a vector of positive measures. In the classical case of one compactly supported measure it is well-known that the weak limit of the Christoffel-Darboux kernel on the diagonal and of the normalized zero counting measure are the same. Under some mild conditions we shall prove an extension of this statement to the general vector case. In the proof we study the Hessenberg matrix corresponding to multiple orthogonal polynomials along any ray of multi-indices tending to infinity. This is a joint work with Van Assche (KU Leuven).

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RODRIGUES FORMULA FOR ORTHOGONAL AND MULTIPLE ORTHOGONAL POLYNOMIALS

MIKHAIL TYAGLOV

The Rodrigues formula is a very important tool to study orthogonal polynomials. In the talk, we discuss a general scheme to find the Rodrigues formula for a few classes of continues and discrete orthogonal and multiple orthogonal polynomials, as well as the correspondent differential and difference equations for them.

This is a joint work with Alexander Dyachenko and Vladimir Lysov (KIAM).

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ON HERMITE-PADÉ APPROXIMANTS FOR A PAIR OF CAUCHY TRANSFORMS WITH OVERLAPPING SYMMETRIC SUPPORTS

MAXIM YATTSELEV

Abstract

Hermite–Padé approximants of type II are vectors of rational functions with common denominator that interpolate a given vector of power series at infinity with maximal order. We are interested in the situation when the approximated vector is given by a pair of Cauchy transforms of smooth complex measures supported on the real line. The convergence properties of the approximants are rather well understood when the supports consist of two disjoint intervals (Angelesco systems) or two intervals that coincide under the condition that the ratio of the measures is a restriction of the Cauchy transform of a third measure (Nikishin systems). In this work we consider the case where the supports form two overlapping intervals (in a symmetric way) and the ratio of the measures extends to a holomorphic function in a region that depends on the size of the overlap. We derive Szegő-type formulae for the asymptotics of the approximants, identify the convergence and divergence domains (the divergence domains appear for Angelesco systems but are not present for Nikishin systems), and show the presence of overinterpolation (a feature peculiar for Nikishin systems but not for Angelesco systems).

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LARGE n LIMIT FOR THE PRODUCT OF TWO COUPLED RANDOM MATRICES

LUN ZHANG

Abstract

For a pair of coupled rectangular random matrices we consider the squared singular values of their product, which form a determinantal point process. We show that the limiting mean distribution of these squared singular values is described by the second component of the solution to a vector equilibrium problem and also establish the local universality. The proof is based on the steepest descent analysis for a 4×4 matrix-valued Riemann-Hilbert problem, which characterizes the correlation kernel and is related to mixed type multiple orthogonal polynomials associated with the modified Bessel functions. Joint work with Guilherme L. F. Silva.

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SECTION 3 : MINISYMPOSIUM SPEAKERS

#7 Multivariable special functions and representation theory

ZETA DISTRIBUTIONS AND RELATED BESSEL FUNCTIONS IN THE DUNKL SETTING

DOMINIK BRENNECKEN

Abstract

The Dunkl setting related to root systems of type A generalizes many classical results from the radial analysis on symmetric cones. Concepts such as the Laplace transform, Riesz distributions, spherical functions and hypergeometric functions have a natural extension to the Dunkl setting. Based on this, we shall introduce in this talk generalizations of J- and K-Bessel functions on symmetric cones to the Dunkl setting. We will show that these generalized Bessel functions share important properties with their analogues on symmetric cones. Further, we shall present some analytic aspects of an associated Hankel transform which goes back to Baker, Forrester and Macdonald. Base on the fact that the J-Bessel function is given by a type B Dunkl-Bessel function, we shall study zeta integrals and zeta distributions. We shall prove recurrence properties and a functional equation which is in line with the properties of zeta distributions on symmetric cones. Finally, we analyse which of these zeta distributions are positive measures.

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AFFINE PIERI RULE FOR PERIODIC MACDONALD SPHERICAL FUNCTIONS AND FUSION RINGS

ERDAL EMSIZ

Abstract

Let $\hat{\mathfrak{g}}$ be an untwisted affine Lie algebra or the twisted counterpart thereof (which excludes the affine Lie algebras of type $\widehat{BC}_n = A_{2n}^{(2)}$). We present an affine Pieri rule for a basis of periodic Macdonald spherical functions associated with $\hat{\mathfrak{g}}$. In type $\hat{A}_{n-1} = A_{n-1}^{(1)}$ the formula in question reproduces an affine Pieri rule for cylindric Hall-Littlewood polynomials due to Korff, which at $t = 0$ specializes in turn to a well-known Pieri formula in the fusion ring of genus zero $\widehat{\mathfrak{sl}}(n)_c$-Wess-Zumino-Witten conformal field theories.

Based on joint work with J. F. van Diejen and I. N. Zurrián.

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HEAT KERNELS FOR W-INVARIANT DUNKL LAPLACIANS

PIOTR GRACZYK

Abstract

In two joint papers with Patrice Sawyer, we prove exact estimates for the W-invariant Dunkl heat kernel, for the root system of type A with arbitrary positive multiplicities. We apply the estimates of the W-invariant Dunkl heat kernel to compute sharp estimates for the Newton kernel and for the s-stable semigroups generated by a fractional power of the W-invariant Dunkl Laplacian.

We also find exact estimates in three variables x, y, t of the heat kernel $h_t(x, y)$ in the curved complex case $k = 1$ for the root systems A, in particular for the 3-dimensional real hyperbolic space $\mathrm{H}^3(\mathbf{R}^d)$. We formulate a conjecture for the general Dunkl rational and trigonometric cases.

References

[1] P. Graczyk and P. Sawyer. *Sharp Estimates of Radial Dunkl and Heat Kernels in the Complex Case A_n*, Comptes Rendus Mathématiques, 359(2021), 427–437.

[2] P. Graczyk and P. Sawyer. *Sharp estimates for W-invariant Dunkl and heat kernels in the A_n case* (2021), arXiv:2111.13529, 1–19.

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ASYMPTOTICS OF THE DIMENSION OF THE DIAGONAL COINVARIANT RING OF A COMPLEX REFLECTION GROUP

STEPHEN GRIFFETH

Abstract

We report on recent results and conjectures on the dimension of the quotient ring by the diagonal invariants for a complex reflection group. Partly based on joint work with Carlos Ajila.

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MULTIVARIATE ASKEY-WILSON FUNCTIONS

WOLTER GROENEVELT

Abstract

Askey-Wilson functions occur as overlap coefficients for eigenvectors of twisted primitive elements in principal series representations of $U_q(su(1,1))$. Multivariate versions of the Askey-Wilson functions have a similar interpretation in tensor products of $U_q(su(1,1))$. This leads to an algebraic construction of a family of commuting q-difference operators for which the multivariate Askey-Wilson functions are eigenfunctions.

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BERNSTEIN-SZEG MEASURES IN THE PLANE

PLAMEN ILLIEV

Abstract

I will discuss spectral properties and characterizations of bivariate measures which can be considered as natural extensions of the Bernstein-Szegö measures on the real line. Based on joint work with J. Geronimo.

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GELFAND PAIRS AND INJECTIVITY OF SPHERICAL AVERAGES ON H-TYPE GROUPS

E. K. NARAYANAN

Abstract

Let N be an H-type group identified with its Lie algebra $V \oplus \mathfrak{z}$ where $\mathfrak{z}$ is the center of N and $[V, V] = \mathfrak{z}$. Let μ_r be the normalized surface measure on the sphere of radius $r > 0$ centered at the origin in V. We show that the spherical mean operator $f \to f * \mu_r$ is injective on $L^p(N)$ for $1 \leq p \leq \frac{2m}{m-1}$, where m is the dimension of $\mathfrak{z}$. If U is the closed subgroup of the group of orthogonal automorphisms of N which act trivially on $\mathfrak{z}$, then it is known that $L^1_U(N)$ (the algebra of U-invariant integrable functions) is commutative (that is (N, U) is a Gelfand pair) when $m = 1, 2$ or 3 ($m = 1$ corresponds to the Heisenberg group). We study the averages defined by the normalized measure on orbits of U in V. When $m = 3$, this differs from the spherical means considered above and we prove the injectivity of these averaging operators for the range $1 \leq p \leq 3$. Spectral decomposition in terms of Laguerre functions and harmonic analysis on nilpotent groups play a key role in the proofs.

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STOCHASTIC PARTICLE SYSTEMS FROM SYMMETRIC FUNCTIONS

LEONID PETROV

Abstract

Most stochastic particle systems which are integrable (i.e., admit explicit formulas powering their asymptotic analysis) are connected to various families of symmetric functions. I will survey a number of identities coming from symmetric functions (Cauchy identity, branching rule, Robinson-Schensted-Knuth type correspondences) and explain how they lead to integrability in interesting stochastic particle systems like exclusion processes, stochastic vertex models, and lozenge tilings.

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VECTOR-VALUED HECKMAN-OPDAM POLYNOMIALS – A STEINBERG VARIATION

MAARTEN VAN PRUIJSSEN

Abstract

We view Heckman-Opdam polynomials as a family of vector-valued polynomials whose members are uniquely determined (up to scaling) as eigenfunctions of a commutative algebra of differential operators. The initial data is a root system and a multiplicity function. We propose a version of this theory relative to a reflection subgroup W' of the Weyl group W associated to the root system. We obtain a family of vector-valued orthogonal polynomials whose members are uniquely determined (up to scaling) as eigenfunctions of a commutative algebra of differential operators. If W'=W we retain the Heckman-Opdam polynomials. Some families of vector-valued orthogonal polynomials from the literature can also be understood in this context.

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MATRIX ORTHOGONAL POLYNOMIALS, DUALITY AND DIFFERENCE OPERATORS

PABLO ROMAN

Abstract

In this talk, we will discuss a notion of duality for matrix valued orthogonal polynomials with respect to a measure supported on the nonnegative integers. We will describe the relation between the dual families and certain difference operators acting on the matrix polynomials. We will also investigate how the properties of the matrix orthogonal polynomials, such as recurrence relations, lowering and rising relations and orthogonality, are translated to their duals.

In order to illustrate duality, we will discuss a family of Charlier type matrix orthogonal polynomials with explicit duals.

This is a joint work with Bruno Eijsvoogel and Luca Morey.

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SHARP ESTIMATES FOR THE HYPERGEOMETRIC FUNCTIONS ASSOCIATED TO ROOT SYSTEMS

PATRICE SAWYER

Abstract

In three papers, joint with Piotr Graczyk, we conjecture exact estimates for the Weyl-invariant hypergeometric functions in the Dunkl and Opdam-Cherednik settings. In this talk, we give an outline of the proof of the conjecture for the root systems of type A. We provide other evidence that the conjecture might be true in general.

References

[1] P. Graczyk and P. Sawyer. *Sharp Estimates of Radial Dunkl and Heat Kernels in the Complex Case A_n*, Comptes Rendus Mathématiques, 359(2021), 427–437.

[2] P. Graczyk and P. Sawyer. *Sharp estimates for W-invariant Dunkl and heat kernels in the A_n case* (2021), arXiv:2111.13529, 1–19.

[3] P. Graczyk and P. Sawyer. *Sharp estimates for the hypergeometric functions related to root systems of type A and of rank 1* (2022), arXiv: arXiv:2203.10025, 1–13.

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QUASI-POLYNOMIAL EXTENSIONS OF MACDONALD POLYNOMIALS

JASPER STOKMAN

Abstract

In this talk I report on recent joint work with Siddhartha Sahi and Vidya Venkateswaran (arXiv:2204.13729) in which we introduce a common generalisation of the nonsymmetric Macdonald polynomials and the metaplectic Whittaker functions. The resulting family of functions are no longer Laurent polynomials but quasi-polynomials, meaning that the monomials are allowed to have non-integral exponents. Many properties of nonsymmetric Macdonald polynomials have a natural counterpart for these quasi-polynomials, some of which will be highlighted in the talk. In particular we highlight the representation theoretic context involving the double affine Hecke algebra.

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HARISH-CHANDRA INTEGRALS FOR HYPERGEOMETRIC FUNCTIONS OF TYPE BC AND LIMIT THEOREMS FOR ASSOCIATED RANDOM WALK

MICHAEL VOIT

Abstract

The Heckman-Opdam hypergeometric functions of type BC in q variables depend on 3 multiplicity parameters; they generalize the spherical functions of the non-compact Grassmann manifolds of rank q over the real, complex, and quaternionic numbers. For these 3 discrete geometric families, the explicit product formulas and the Harish-Chandra integral representations for these functions were extended by interpolation to 3 continuous families of parameters by Rösler (2010) such that the hypergeometric functions are the multiplicative functions of associated commutative hypergroup structures on the closed Weyl chambers of type B.

We show how these Harish-Chandra integrals can be used to study so-called moment functions which appear as partial derivatives w.r.t. the spectral parameters at the identity character. These results lead to strong laws of large numbers and central limit theorems for random walks which are related with these hypergroup structures. In the geometric cases, these results have interpretations for the non-compact Grassmann manifolds.

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SECTION 3 : MINISYMPOSIUM SPEAKERS

#8 PhD student and postdoctoral fellow Minisymposium

PADÉ APPROXIMANTS FOR FUNCTIONS WITH FOUR BRANCH POINTS

AHMAD BARHOUMI

Abstract

This talk centers around the large-degree behavior of non-Hermitian orthogonal polynomials arising from Padé approximants (or "best" rational approximants). In particular, I will discuss approximants corresponding to a class of functions with four branch points. While it might seem modest, four branch points are already enough to generate certain singular, symmetric, arrangements where the orthogonal polynomials behave unexpectedly. I will highlight how these difficulties appear in the analysis and how one might remedy them. This talk is based on joint work with Maxim Yattselev.

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ON THE ZEROS OF HANKEL TRANSFORM

SEOK-YOUNG CHUNG

Abstract

Inspired by Polya's monumental work (1918), we extended the Polya's result for the zeros of Fourier transform to that for the zeros of Hankel transform.

In this talk, we consider the Hankel transform of the integrable function on [0,1], and will mainly discuss the distribution of the zeros of Hankel transform by means of the partial fraction expansions in terms of Hankel transform. To be precise, the zeros of Hankel transform are simple and real in accordance with certain condition for function being transformed, indeed the positive zeros of Hankel transform are interlaced with the positive zeros of Bessel function or those of Hankel transform of different order.

This talk is based on joint work with Yong-Kum Cho and Young Woong Park.

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$N \times N$ MATRIX TIME–BAND LIMITING EXAMPLES

BRUNO EIJSVOOGEL

Abstract

Noncommutative time–band limiting could shed light on the miraculous commutativity between certain differential and integral operators that appear in classical time–band limiting.

Certain types of matrix-valued orthogonal polynomials provide explicit examples that fit into the noncommutative time–band limiting framework. We will show that matrix weight functions that satisfy strong Pearson equations fit more easily into this framework.

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STOCHASTIC FACTORIZATIONS OF BIRTH AND DEATH CHAINS AND DARBOUX TRANSFORMATIONS

CLAUDIA IVONNE JUAREZ GALLEGOS

Abstract

The study of Markov chains using orthogonal polynomials began in the 1950s when S. Karlin and J. McGregor published a series of articles in which they analyzed birth-death chains with state space in . Within this context we study different ways of stochastic factorizations of the transition probability matrix of a discrete-time birth-death chain on the integers, which is a doubly infinite tridiagonal stochastic Jacobi matrix. We give conditions such that each of the factors is a stochastic matrix. By performing a discrete Darboux transformation we get new families of birth-death chains on the integers and we identify the corresponding spectral matrices which are conjugations of the original one. This is a join work with M. D. de la Iglesia.

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SOME PROPERTIES AND APPLICATIONS OF EXTENDED HYPERGEOMETRIC FUNCTIONS

RAHUL GOYAL

Abstract

Euler's beta, and gamma functions (hypergeometric functions) are the important members of the family of special functions and it plays a vital role in the whole theory of special functions. These hypergeometric functions together with their extension have many applications in research fields such as engineering, chemical, statistics, fractional calculus, and physical problems. In this Symposium, our discussions have been focused on the extended Euler's beta function, which is developed by using the 2-parameter Mittag-Leffler function as the kernel. We discuss various basic properties and formulas of the extended Euler's beta function such as integral representations, transformation formulas, and summation formulas. We also introduce the logarithmic convexity and some important inequalities for this extended Euler's beta function. Then by using this extended Euler's beta function as kernel, we have generalized hypergeometric functions and study various properties of these extended hypergeometric functions. From an application point of view, we have also derived some relations between this extended Euler's beta function and extended fractional derivative operators such as Caputo fractional derivative operator and Riemann-Liouville fractional derivative operator.

Keywords: Euler's beta function, gamma function, hypergeometric function, Mittag-Leffler function, Caputo fractional derivative operator, and Riemann-Liouville fractional derivative operator.

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SOME RECENT THEORETICAL RESULTS ABOUT HILBERT SPACE EMBEDDINGS OF PROBABILITIES

JEAN CARLO GUELLA

Abstract

We review some recent theoretical results about Hilbert space embeddings of probabilities, like the fact that Gaussian kernels on Hilbert spaces defines an inner product in the space of measures with bounded variation and that the standard metric in Hilbert and real/complex hyperbolic spaces are of strong negative type. We also present the concept of positive definite independent kernels, which generalizes the concepts of Hilbert Schmidt Independence Criterion and Distance Covariance, provides a metric in the space of couplings, and are related to Bernstein functions with 2 variables.

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KOSHLIAKOV ZETA FUNCTIONS AND MODULAR RELATIONS

RAJAT GUPTA

Abstract

Nikolai Sergeevich Koshliakov was an outstanding Russian mathematician who made phenomenal contributions to number theory, special functions, and differential equations. In the aftermath of World War II, he was one among the many scientists who were arrested on fabricated charges and incarcerated. Under extreme hardships while still in prison, Koshliakov (under a different name 'N. S. Sergeev') wrote two manuscripts out of which one was lost. Fortunately the second one was published in 1949 although, to the best of our knowledge, no one studied it until the last year when Prof. Atul Dixit and I started examining it in detail. This manuscript contains a complete theory of two interesting generalizations of the Riemann zeta function having their genesis in heat conduction and is truly a masterpiece! In this talk, we will discuss some of the contents of this manuscript and then proceed to give some new results (modular relations) that we have obtained in this theory. This is a joint work with Professor Atul Dixit.

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EXTENDED HIGHER HERGLOTZ FUNCTIONS

RAHUL KUMAR

Abstract

In 1975, Don Zagier obtained the Kronecker limit formula for a real quadratic field which involved an interesting function $F(x)$ he termed Herglotz function. As demonstrated by Zagier, and very recently by Radchenko and Zagier, $F(x)$ satisfies beautiful properties which are of interest to both algebraic as well as analytic number theorists. In this talk, we will discuss $F_{k,N}(x)$, an extension of the Herglotz function which also subsumes higher Herglotz function of Vlasenko and Zagier. We obtain functional equations satisfied by $F_{k,N}(x)$. Radchenko and Zagier gave a beautiful relation between an integral and $F(x)$ and used it to evaluate that integral at various rational as well as irrational arguments. We obtain a relation involving a generalization of the above integral and $F_{k,N}(x)$. This is joint work with Professor Atul Dixit and Rajat Gupta.

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ON THE q-RIEMANN HILBERT PROBLEM AND ITS APPLICATION TO q-ORTHOGONAL POLYNOMIALS

TOMAS LASIC LATIMER

Abstract

The theory of Riemann-Hilbert Problems (RHP) provides a major tool for investigating the asymptotic behaviours of certain classes of orthogonal polynomials. However, there remain classes of q-orthogonal polynomials for which the methodology has not been developed. In this talk, we extend the approach to a class of polynomials that are orthogonal with respect to a compact q-lattice, i.e. q^k for $k \in \mathbb{N}$, and deduce their asymptotic behaviours from the corresponding RHP in the limit their degree tends to infinity. Our results rely on a finite sequence of transformations, ending with a model RHP which is solvable. If time permits, we will also describe the extension to polynomials orthogonal with respect to a non-compact q-lattice, i.e. q^k for $k \in \mathbb{Z}$.

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MATRIX WEIGHT FOR MULTIVARIABLE ORTHOGONALITY FROM GROUP REPRESENTATIONS

JIE LIU

Abstract

The topic of this talk is to give an explicit $S(U(n)\times U(m))$-intertwiner j from a specific irreducible $S(U(n)\times U(m))$-module to an irreducible $SU(n+m)$-module. It is an intermediate step for calculating an important matrix-valued function on symmetric pair $(SU(n+m), S(U(n)\times U(m)))$, i.e. matrix spherical function. This function is a matrix-valued multivariable function, which is a generalization of the Gegenbauer polynomials for the case $m = n = 1$. We find an $S(U(n)\times U(m))$-intertwiner from an irreducible $S(U(n)\times U(m))$-module to a bigger $SU(n+m)$-module using the Laplace expansion at first, then we calculate j explicitly using Schur orthogonality. This gives possibilities to calculate the matrix weight for these multivariable matrix orthogonal polynomials.

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A NEW APPROACH TO LOCAL AND GLOBAL STATISTICS FOR THE ELLIPTIC GINIBRE ENSEMBLE IN HIGHER DIMENSIONS

LESLIE MOLAG

Abstract

The complex elliptic Ginibre ensemble allows one to interpolate between the Ginibre ensemble and the Gaussian Unitary ensemble. It represents a determinantal point process in the complex plane with corresponding kernel, constructed with planar Hermite polynomials. Our main tool is a saddle point analysis of a single contour integral representation of this kernel. It provides a unifying approach to rigorously derive several known and new results of local and global spectral statistics. In particular, we prove rigorously some global statistics in the elliptic Ginibre ensemble first derived by Forrester and Jancovici. The limiting kernel receives its main contribution from the boundary of the limiting elliptic droplet of support. We introduce a d-complex dimensional generalization of the elliptic Ginibre ensemble, which interpolates between d-real and d-complex dimensions. In the Hermitian limit, this new ensemble is related to non-interacting Fermions in a trap in d-real dimensions with d-dimensional harmonic oscillator. We provide new local bulk and edge statistics at weak and strong non-Hermiticity for this new ensemble.

This is joint work with Gernot Akemann and Maurice Duits.

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AN EXAMPLE OF DUALITY FOR MATRIX VALUED ORTHOGONAL POLYNOMIALS

LUCÍA, MOREY

Abstract

In this talk we discuss the construction of a nontrivial family of matrix orthogonal polynomials with a dual family which is also orthogonal. We say that the sequences $(P_n^{(\lambda)})_n$ and $(Q_x^{(\lambda)})_x$ of matrix polynomials are dual with each other if

$$P_n^{(\lambda)}(x) = P_n^{(\lambda)}(0)Q_x^{(\lambda)}(\rho^{(\lambda)}(n))\Upsilon^{(\lambda)}(x), \qquad \text{where } n, x \in \mathbb{N} \cup \{0\},$$

for certain matrix valued functions $\rho^{(\lambda)}, \Upsilon^{(\lambda)}$. This definition extends Leonard's result [1] to the matrix valued setting. We end this talk with a nice connection between the weight matrices and the squared norms of the polynomials, in the seance that the dual weight $U^{(\lambda)}(n)$ is related to the squared norm $\mathcal{H}_n^{(\lambda)}$ of the polynomials $(P_n^{(\lambda)})_n$ by

$$U^{(\lambda)}(n) = P_n^{(\lambda)}(0)^*(\mathcal{H}_n^{(\lambda)})^{-1}P_n^{(\lambda)}(0),$$

and the squared norm of the dual polynomials $\mathcal{W}_x^{(\lambda)}$ is related to the weight $W^{(\lambda)}$ by

$$\mathcal{W}_x^{(\lambda)} = (\Upsilon^{(\lambda)}(x)W^{(\lambda)}(x)\Upsilon^{(\lambda)}(x)^*)^{-1}.$$

References

[1] Douglas A. Leonard (1982) *Orthogonal polynomials, duality and association schemes*. SIAM J. Math. Anal.

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THE MITTAG-LEFFLER CONFLUENT HYPERGEOMETRIC MATRIX FUNCTION AND THEIR FRACTIONAL CALCULUS

ANKIT PAL

Abstract

In this work, we define a matrix analogue of Mittag-Leffler confluent hypergeometric function and establish several integral representations and derivative formulas. We also define the finite summation formulas for the Mittag-Leffler confluent hypergeometric matrix function. Further we introduce the generalized fractional calculus operators of power function and obtain the composition of Mittag-Leffler confluent hypergeometric matrix function with the generalized fractional calculus operators.

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A CONVERGENT AND ASYMPTOTIC LAPLACE METHOD FOR INTEGRALS

PABLO PALACIOS

Abstract

Watson's lemma and Laplace's method provide asymptotic expansions of Laplace integrals $F(z) := \int_0^\infty e^{-zf(t)}g(t)dt$ for large values of the parameter z. They are useful tools in the asymptotic approximation of special functions that have a Laplace integral representation. But in most of the important examples of special functions, the asymptotic expansion derived by means of Watson's lemma or Laplace's method is not convergent.

In this work, we combine the modified Laplace's method introduced in [1] with a logarithmic change of variables to derive asymptotic expansions of $F(z)$ that are also convergent, accompanied by error bounds. As illustration, a new convergent and asymptotic expansion of the parabolic cylinder function $U(a, z)$ for large $|z|$ is derived.

References

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AREA-DEPTH SYMMETRIC CATALAN POLYNOMIAL

JOSEPH PAPPE

Abstract

We define two new symmetric q,t-Catalan polynomials in terms of area and a new statistic called depth and in terms of dinv and a new statistic called dinv of depth. We prove symmetry using an involution on plane trees from which we obtain another description of the usual q,t-Catalan polynomials. The same involution proves symmetry of the Tutte polynomial for the Catalan Matroid.

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ANALOGUES OF EPSTEIN'S ZETA FUNCTION AND APPLICATIONS

PEDRO RIBEIRO

Abstract

In this talk, we propose a generalization of the classical Selberg-Chowla formula, motivating the study of the analytic continuation of a large class of Epstein zeta functions. We indicate how this formula is useful to study the distribution of zeros of Dirichlet series in their critical lines and to derive analogues of Guinand's and Koshliakov's formulas. We also discuss an existing connection to the Kontorovich-Lebedev transform of Dirichlet series.

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A GENERATING FUNCTION ASSOCIATED WITH THE ALTERNATING ELEMENTS IN THE POSITIVE PART OF $U_q(\widehat{\mathfrak{sl}}_2)$

CHENWEI RUAN

Abstract

The positive part U_q^+ of $U_q(\widehat{\mathfrak{sl}}_2)$ admits an embedding into a q-shuffle algebra. This embedding was introduced by M. Rosso in 1995. In 2019, Terwilliger introduced the alternating elements $\{W_{-n}\}_{n\in\mathbb{N}}$, $\{W_{n+1}\}_{n\in\mathbb{N}}$, $\{G_n\}_{n\in\mathbb{N}}$, $\{\tilde{G}_n\}_{n\in\mathbb{N}}$ in U_q^+ using the Rosso embedding. He showed that the alternating elements $\{W_{-n}\}_{n\in\mathbb{N}}$, $\{W_{n+1}\}_{n\in\mathbb{N}}$, $\{\tilde{G}_{n+1}\}_{n\in\mathbb{N}}$ form a PBW basis for U_q^+. He also studied the generating function $\tilde{G}(t)$ for $\{\tilde{G}_n\}_{n\in\mathbb{N}}$ as well as the multiplicative inverse $D(t)$ of $\tilde{G}(t)$. In this talk, we will express the coefficients of $D(t)$ in closed form.

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APPLICATIONS OF THE REMEZ ALGORITHM TO CHEBYSHEV POLYNOMIALS IN THE COMPLEX PLANE

OLOF, RUBIN

Abstract

The Chebyshev Polynomial of degree n corresponding to an infinite compact set $\mathbb{E} \subset \mathbb{C}$ is the unique monic polynomial of degree n which minimizes the sup-norm on the set. Using the Complex Remez Algorithm introduced by P. Tang and then further developed by J. Modersitzki and B. Fischer we are able to compute Chebyshev polynomials corresponding to a large variety of sets. This enables us to get a concrete point of view of how these polynomials behave, regarding sup-norm and zero placement. In this talk I will present plots related to these computed polynomials and compare these to theoretical results and further discuss how these plots can be used to formulate new conjectures based on numerical evidence. The work is based on joint work with J. S. Christiansen.

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AN ASKEY-WILSON ALGEBRA OF RANK 2

CAREL, WAGENAAR

Abstract

In this talk a rank 2 extension of the Askey-Wilson algebra is introduced. We will show that bivariate q-Racah polynomials appear as overlap coefficients of eigenvectors of generators of this algebra. Also, we explicitly calculate the corresponding q-difference operators. Relations in this algebra are motivated by relations between coproducts of elements in the tensor product of the quantum algebra $\mathcal{U}_q(\mathfrak{sl}(2,\mathbb{C}))$.

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ORTHOGONAL RATIONAL FUNCTIONS WITH REAL POLES, ROOT ASYMPTOTICS, GMP MATRICES

GIORGIO YOUNG

Abstract

There is a vast theory of the asymptotic behavior of orthogonal polynomials with respect to a measure on $\mathbb{R}$ and its applications to Jacobi matrices. That theory has an obvious affine invariance and a very special role for ∞. I will discuss a paper which extends aspects of this theory in the setting of rational functions with poles on $\overline{\mathbb{R}} = \mathbb{R} \cup \{\infty\}$, obtaining a formulation which allows multiple poles and proving an invariance with respect to $\overline{\mathbb{R}}$-preserving Möbius transformations. In this paper, we obtain a characterization of Stahl–Totik regularity of a GMP matrix in terms of its matrix elements; as an application, we give a proof of a conjecture of Simon – a Cesàro–Nevai property of regular Jacobi matrices on finite gap sets. This is joint work with Benjamin Eichinger and Milivoje Lukić.

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SECTION 3 : MINISYMPOSIUM SPEAKERS

#9 General session

A NEW APPROACH TO EVALUATING MALMSTENS INTEGRAL AND RELATED INTEGRALS

ABDULHAFEEZ AYINDE ABDULSALAM

Abstract

This paper discusses a generalization of logarithmic integrals and integrals involving hyperbolic functions. A new proof for an integral presented by Vardi, and several generalizations of integrals involving certain combinations of transcendental functions in relation to known mathematical constants are discovered. We derive a reduction formula for Malmstens integral with a generalized power of the hyperbolic secant function, and we establish the role of the complete Bell polynomials in generalizing the integral. A new generalization of a class of integrals involving hyperbolic functions is also discovered.

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CONDITIONAL EXPANDING OF FUNCTIONS BY q-LIDSTONE SERIES

MARYAM A. AL-TOWAILEB

Abstract

In this paper, we will provide a characterization of those functions given by convergent q-Lidstone series expansion. We give the conditions which are necessary and sufficient so that the entire function $f(z)$ has such an expansion in which case convergence is uniform on compact sets.

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CORRECTION OF AN IDENTITY INVOLVING KUMMER'S FORMULA

MOHAMED JALEL ATTIA

Abstract

Abstract. In the handbook "mathematical functions with formulas, graphs and mathematical tables" done by Abramowitz-Stegun (resp. in the book "special functions" done by G. Andrews, R. Askey and R. Roy), the identity 15.3.20 (resp. 3.1.7 page 127 with a slight modification) given by

$$ {}_2F_1(a, a+\frac{1}{2}; c; z) = \frac{1}{(1 \pm \sqrt{z})^{2a}}\ {}_2F_1(2a, c-\frac{1}{2}; 2c-1; \pm\frac{2\sqrt{z}}{1 \pm \sqrt{z}}) $$

is not true for every value of a and c. In this paper, we prove that the identity 15.3.20 is false for any positive integer a and $c = -n + \frac{3}{2},\ n \geq 2$ (n integer) and should be written as

$$ \begin{aligned} {}_2F_1(a, a+\frac{1}{2}; -n+\frac{3}{2}; z^2) &= \frac{1}{(1 \pm z)^{2a}}\ {}_2F_1(2a, -n+1; -2n+2; \frac{\pm 2z}{1 \pm z}) \\ &\pm \frac{2\sqrt{\pi}\Gamma(n+a) z^{2n+2a-3}\ {}_2F_1(1-a, -n-a+\frac{3}{2}; \frac{3}{2}; \frac{1}{z^2})}{\Gamma(a)\Gamma(n-\frac{1}{2})(z^2-1)^{n+2a-1}},\ n \geq 2. \end{aligned} $$

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SOME EXAMPLES OF THE NEW MATRIX q-LIE ALGEBRAS

THOMAS ERNST

Abstract

We consider several special cases of the new concept of a q-Lie algebras. First we define such a q-Lie algebra together with its Cartan subalgebras, q-root system, Killing form and weight lattice. The connection with q-special functions comes from a maximal q-torus contained in the associated matrix q-Lie group, and leads to q-trigonometric and q-hyperbolic functions. In this way, the q-addition replaces ordinary addition; actually there are several forms of q-addition, which can be found in the authors books and articles [1].

References

[1] T. Ernst, A comprehensive treatment of q-calculus, Birkhauser (2012)

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BIVARIATE AND MULTIVARIATE BETA-POCHHAMMERS: AN ENGINEERING APPROACH TO INCOMPLETE HYPERGEOMETRIC FUNCTIONS WITH APPLICATIONS TO PROBABILITY DISTRIBUTIONS

JOLYON DE FREITAS

Abstract

Since the first description of the incomplete Pochhammer symbol by Srivastava and colleagues [1], there has been an upsurge in activities describing incomplete hypergeometric functions of one or more variables and their applications. This is a single indexed symbol $(a, z)_n = \gamma(a + n, z)/\Gamma(a)$, $n \in \mathbb{N}, R(a) > 0; z \geq 0$, where $\gamma(a + n, z)$ is the incomplete Gamma function.

In tackling an unresolved engineering probability distribution describing the phase of a vector perturbed by Gaussian noise, it was discovered that the incomplete Beta function can be set up in a similar manner with a double indexed symbol $(a, b; z)_{(m,n)} = B_z(a + m, b + n)/B(a, b)$, $m, n \in \mathbb{N}$, $R(a) > 0; R(b) > 0; z \in (0, 1)$, where $B_z(a + m, b + n)$ and $B(a, b)$ are the incomplete and complete Beta function, respectively. This rising factorial representation of two variables has been called the incomplete Beta-Pochhammer, forming the basis of closed-form solutions to the probability phase error and the cumulative distribution function.

This presentation will look at the incomplete bivariate Beta-Pochhammer and will also introduce the incomplete multivariate Beta-Pochhammer symbol, closed-form sums and their engineering interpretations. We consider the wider discussion of incomplete single and multivariate hypergeometric functions and propose a generalised incomplete Selberg-type integral of several variables.

References

[1] Integral Transforms Spec. Funct. 23 (2012) 659-683

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THE INCOMPLETE JACOBI ELLIPTIC FUNCTIONS IN A MATHEMATICAL PHYSICS PROBLEM AND IN A MATHEMATICAL BIOLOGY PROBLEM

OLIVIER LAFITTE

Abstract

The incomplete Jacobi function K appears in explicit solutions in two new problems.

The first problem is the coupling between the stationary neutronics diffusion equation and the thermohydraulics systems of equations. Under simple but relevant forms of the coefficients of the neutronics equation, one obtains the ODE for the Jacobi equation and this analysis provides much more precise results than most of the numerical methods used for coupling codes. Moreover, the algegraic structure of the underlying elliptic curves yields a nice description of the solution in three general case.

The second problem deals with the electrostatics in ions in a biological cell. This problem couples the electrostatic diffusion equation for the electrical potential in the cell with the spreading of the ions inside this cell, modelled by a diffusion equation for the density of each species. In a 1d set-up, this system boils down to a Jacobi equation for the density of ions, and one deduces the electrostatic potential as well as the boundary layer where the ions are present, giving a new interpretation to the Gouy-Chapman boundary layer.

In both problems, such explicit solutions are extremely useful for studying precisely the dependecy of solutions with respect to the physical parameters.

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SPECIAL FUNCTIONS AND THEORY OF DISTRIBUTIONS

ASIFA TASSADDIQ

Abstract

The combination of distributions (generalised functions) and integral transformations proved an important tool to crack a variety of unsolvable problems. Getting inspired by this fact, this research comprises a novel method to represent special functions by developing the basic concepts of distribution theory for them. More specifically, an extension of the domain of validity of gamma, beta, zeta, hypergeometric, Fox-Wright, and Fox-H functions is proved by defining them as a series of delta functions. Laplace transform of various well-studied special functions is also determined. It proved significant for the formulation and solution of new fractional kinetic equations. It can be stated that this research article is a new confluence of two mathematical disciplines, namely special functions and distributions (generalised functions).

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APPLICATIONS OF ORTHOGONAL POLYNOMIALS IN PARALLEL COMPUTATION AND NUMERICAL INTEGRATION

XIANG-SHENG WANG

Abstract

There are two challenging problems in parallel numerical computation for an initial value problem. The first one is to design a suitable time discretization matrix that is diagonalizable. The other one is to estimate the condition number of the corresponding eigenvector matrix. In the first part of this talk, we will make an innovative application of orthogonal polynomials in tackling these problems. In the second part of the talk, we will prove Harris-Simanek's conjecture on bivariate Lagrange interpolation related to numerical integration.

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