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# In every finite-range quantum spin chain, linearised Euler hydrodynamic equations hold

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Euler hydrodynamics is a set of equations that describe the dynamics of many-body systems at long times and large wavelengths. It is extremely powerful, as it necessitates only a few emergent degrees of freedom, instead of the full microscopic dynamics of a large number of particles, spins or fields. It is extremely general, as it applies in principle to classical and quantum systems of many types. But rigorously establishing that it indeed emerges, and identifying the correct degrees of freedom, are some of the most important challenges of mathematical and theoretical physics. I will present in a pedagogical way what I believe are the first general rigorous results in this direction. The linearised Euler hydrodynamic equations, describing two-point correlation functions in statistical ensembles, are established in every quantum spin chain with finite-range interaction. This hold no matter the specific properties of the chain: integrable, chaotic, constrained, etc. The exact set of emergent degrees of freedom — the extensive conserved quantities — is identified as a certain Hilbert space. In integrable spin chains, it is infinite-dimensional and includes the quasi-local charges found in the context of generalised thermalisation. For the simple example of free-fermion chains, the principles of generalised hydrodynamics — the hydrodynamics of integrable systems — are recovered. Interestingly, the mathematical theory easily extends to oscillatory behaviours, showing that hydrodynamic ideas can be applied beyond the conventional low-frequency, large-wavelength region.