Conférenciers mini-cours - 23-27 mai 2022
Mini-courses speakers - May 23-27, 2022

Georgia Benkart (University of Wisconsin-Madison)

With great sadness, we inform you that Prof. Benkart passed away on April 29, 2022. She had a remarkable career as an algebraist and mentor to young mathematicians.Here are her notice on wiki and a short review of her work published in the Notices of the AMS (March 2022).

Sujet/ Topic: *Walking On Graphs*

The topics for the 3 lectures are:
1. *The McKay Correspondence and Invariants*
2. *Chip-firing and Discrete Dynamical Systems*
3. *Tensor Categories and Fusion Products*

Maud De Visscher (City, University of London)

Titre/ Title: *Representation theory of centraliser algebras*

Résumé/Abstract:
In this mini-course, I will present some techniques which can be used uniformly to study the representation theory of centraliser algebras, such as the Temperley-Lieb, Brauer, walled Brauer and partition algebras. I will also explain how we can understand the representations using a reflection geometry and corresponding Kazhdan-Lusztig polynomials.

Pavel Etingof (MIT)

Sujet/Topic: *Symmetric tensor categories*
Titre/ Title: *Algebra and representation theory without vector spaces (lecture 1 and 2)*

Résumé/Abstract:
A modern view of representation theory is that it is a study not just of individual representations (say, finite dimensional representations of an affine group or, more generally, supergroup scheme G over an algebraically closed field k) but also of the category Rep(G) formed by them. The properties of Rep(G) can be summarized by saying that it is a symmetric tensor category (shortly, STC) which uniquely determines G. A STC is a natural home for studying any kind of linear algebraic structures (commutative algebras, Lie algebras, Hopf algebras, modules over them, etc.); for instance, doing so in Rep(G) amounts to studying such structures with a G-symmetry. It is therefore natural to ask: does the study of STC reduce to group representation theory, or is it more general? In other words, do there exist STC other than Rep(G)? If so, this would be interesting, since algebra in such STC would be a new kind of algebra, one "without vector spaces". Luckily, the answer turns out to be ``yes". I will discuss examples in

characteristic zero and $p>0$, and also Deligne's theorem, which puts restrictions on the kind of examples one can have.

Titre/ Title: *Representation theory in non-integral rank (lecture 3)*

Résumé/Abstract:
Examples of symmetric tensor categories over complex numbers which are not representation categories of supergroups were given by Deligne-Milne in 1981. These very interesting categories are interpolations of representation categories of classical groups GL(n), O(n), Sp(n) to arbitrary complex values of n. Deligne later generalized them to symmetric groups and also to characteristic p, where, somewhat unexpectedly, one needs to interpolate n to p-adic integer values rather than elements of the ground field. I will review some of the recent results on these categories and discuss algebra and representation theory in them.

Titre/ Title: *Symmetric tensor categories of moderate growth and modular representation theory (lecture 4)*

Résumé/Abstract:
Deligne categories discussed in Lecture 3 violate an obvious necessary condition for a symmetric tensor category (STC) to have any realization by finite dimensional vector spaces (and in particular to be of the form Rep(G)): for each object X the length of the n-th tensor power of X grows at most exponentially with n. We call this property ``moderate growth". So it is natural to ask if there exist STC of moderate growth other than Rep(G). In characteristic zero, the negative answer is given by the remarkable theorem of Deligne (2002), discussed in Lectures 1,2. Namely Deligne's theorem says that a STC of moderate growth can always be realized in supervector spaces. However, in characteristic p the situation is much more interesting. Namely, Deligne's theorem is known to fail in any characteristic $p>0$. The simplest exotic symmetric tensor category of moderate growth (i.e., not of the form Rep(G)) for $p>3$ is the semisimplification of the category of representations of Z/p, called the Verlinde category. For example, for $p=5$, this category has an object X such that $X^2=X+1$, so X cannot be realized by a vector space (as its dimension would have to equal the golden ratio). I will discuss some aspects of algebra in these categories, in particular failure of the PBW theorem for Lie algebras (and how to fix it) and a generalization of Deligne's theorem in characteristic p due to Kevin Coulembier, Victor Ostrik and myself. This generalization allows one to prove new properties of modular representations of finite groups (and, more generally, affine group schemes) which were previously out of reach. I will also discuss a family of non-semisimple exotic categories in characteristic p constructed in my joint work with Dave Benson and Victor Ostrik, and their relation to the representation theory of groups $(Z/p)^n$ over a field of characteristic p.

Joel Kamnitzer (University of Toronto)

Titre/Title: Webs and Howe duality

Résumé/Abstract: A classic problem in representation theory is to describe the centralizers of tensor products of representations of *gl(n)*. One approach, first proposed by Kuperberg, is to use certain trivalent diagrams, called webs, which generalize Temperley-Lieb diagrams. About 10 years ago, Cautis, Morrison and I realized that we could study webs using *gl(n)*, *gl(m)*-Howe duality. We used this idea to give a complete presentation of the *gl(n)* webs. Since then, these ideas have been applied to other semisimple Lie algebras, as well as to the study of knot invariants and categorification.

Jasper V. Stokman (Universiteit van Amsterdam)

Titre/ Title: *n-point spherical functions and their applications*

Résumé/Abstract:
n-point spherical functions form a special class of vector-valued spherical functions on real reductive Lie groups. They were introduced by the speaker and N. Reshetikhin a couple of years ago, in our (continuing) attempt to obtain a better understanding of n-point correlation functions in WZWN boundary conformal field theories.

In this minicourse I will give an introduction to the theory of n-point spherical functions and explain some of its applications to (multivariate) special functions, integrable systems and quantum topology.