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### **Lie point symmetries and integration of delay ordinary differential equations**

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Lie point symmetries of first-order and second-order delay ordinary differential equations (DODEs) accompanied by an equation for the delay parameter (delay relation) are considered. A subset of such systems (delay ordinary differential systems or DODSs) which consists of linear DODEs and solution independent delay relations have infinite-dimensional symmetry algebras, as do nonlinear ones that are linearizable by an invertible transformation of variables. Genuinely nonlinear first-order DODSs have symmetry algebras of dimension  $n$ ,  $0 \leq n \leq 3$ . Nonlinear second-order DODSs have symmetry algebras of dimension  $n$ ,  $0 \leq n \leq 6$ . A group classification of first-order and second-order delay ordinary differential equations accompanied by an equation for delay relation is developed. It is shown how exact analytical solutions of invariant DODSs can be obtained using symmetry reduction. For the second-order DODSs the Lagrangian formalism was constructed. The Noether-type identity allows to formulate the sufficient conditions for variational second-order DODSs to have first integrals. Several examples are considered.

The talk is based on the following papers in collaboration with Pavel Winternitz:

1. V.A. Dorodnitsyn, R. Kozlov, S. V. Meleshko and P. Winternitz (2018) Lie group classification of first-order delay ordinary differential equations, *J. Phys. A: Math. Theor.* 51 205202.
2. V.A. Dorodnitsyn, R. Kozlov, S. V. Meleshko and P. Winternitz (2018) Linear or linearizable first-order delay ordinary differential equations and their Lie point symmetries, *J. Phys. A: Math. Theor.* 51 205203.
3. V.A. Dorodnitsyn, R. Kozlov, S. V. Meleshko and P. Winternitz (2020) Lie point symmetries of delay ordinary differential equations, in N. Euler and M. Nucci (editors) *Nonlinear Systems and Their Remarkable Mathematical Structures, Volume 2*, Chapman and Hall/CRC, 2020.
4. V.A. Dorodnitsyn, R. Kozlov, S. V. Meleshko and P. Winternitz (2021) Second-order delay ordinary differential equations, their symmetries and application to a traffic problem, *J. Phys. A: Math. Theor.* 54 105204.