Individual and collective fairness in kidney exchange programmes

Péter Biró
Institute of Economics, CERS, and
Corvinus University of Budapest
peter.biro@krtk.mta.hu
joint work with many co-authors

Workshop: Agents behaviour in combinatorial game theory
Montreal (virtual)
16-18 November 2021

Recent papers on kidney exchange programmes

Overviews on **European KEPs** by the ENCKEP COST Action

- P. Biró, Bernadette Haase, and et al.: Building kidney exchange programmes in Europe – an overview of exchange practice and activities. Transplantation, 103 (7): 1514-1522, 2019.
- P. Biró, J. van de Klundert, D. Manlove, and et al.: Modelling and optimisation in European Kidney Exchange Programmes. EJOR, 2020.

Stable exchanges: individual fairness, respecting improvement property

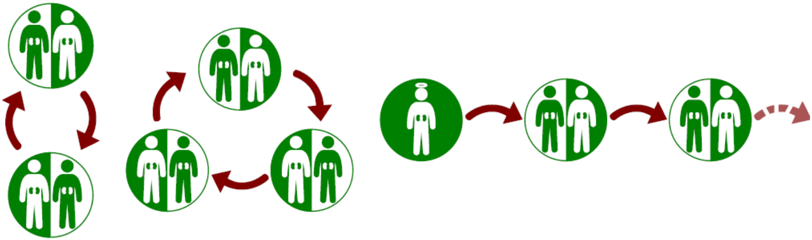
- Klimentova-Biró-Costa-Viana-Pedroso: Novel IP formulations for the stable kidney exchange problem. WP-2021
- Biró-Klijn-Klimentova-Viana: Shapley-Scarf Housing Markets: Respecting Improvement, Integer Programming, and Kidney Exchange. WP-2021
- Schlotter-Biró-Fleiner: The core of housing markets from an agent's perspective: Is it worth sprucing up your home? WINE-2021

Compensation schemes for international KEPs: fairness for countries

- Biró-Kern-Paulusma-Pálvölgyi: Generalized Matching Games for International Kidney Exchange. AAMAS-2019
- Gyetvai-Biró-Klimentova-Pettersson-Viana: Compensation scheme with Shapley value for multi-country kidney exchange programmes. 2020
- Benedek-Biró-Kern-Paulusma: Computing Balanced Solutions for Large International Kidney Exchange Schemes. 2021

Kidney exchange basics

Patients with end-stage renal kidney disease exchange their willing, but immunologically incompatible donors with each other...



pairwise, three-way exchanges, altruistic chains

UK KEP: a pairwise kidney exchange from 2007

Daily Mail, Thursday, December 6, 2007

The transplant pact

Two saved as families exchange kidneys

By Luke Salkeld

THEY were both in desperate need of a kidney donor, and both had relatives who were willing to sacrifice an organ.

But without a family match, strangers Donald Planner and Margaret Wearn instead entered into an extraordinary pact.

Mr Planner's daughter donated her kidney to Mrs Wearn, whose husband gave his kidney to Mr Planner.

The operations took place 170 miles apart in synchronised procedures with the organs transported by ambulances travelling in opposite directions between

'Completely amazing': Donald Planner with his daughter Suzanne

Margaret and Roger Wearn: 'No different to a direct donation'

UK KEP: solutions in early years

Table 1. Results arising from matching runs from April 2008 to October 2009.

Matching run		2008			2009			
		Apr	Jul	Oct	Jan	Apr	Jul	Oct
# pairs		76	85	123	126	122	95	97
# possible donations		287	235	704	576	760	1212	866
Total #	2-cycles	5	2	14	16	20	54	4
	3 cycles	5	0	109	65	68	164	4
Pairwise	#2-cycles	2	1	6	5	5	10	2
exchanges	size	4	2	12	10	10	20	4
	weight	91	6	499	264	388	739	222
$\leq$ 3-way	#2-cycles	2	1	2	1	2	2	0
exchanges	#3-cycles	4	0	7	5	5	9	2
	size	16	2	25	17	19	31	6
	weight	620	6	1122	633	757	1300	300
the exact	size of S	5	0	18	13	14	25	3
algorithm	$\#Y \subseteq S$	24	0	3480	588	1440	67824	6
Running time (sec)		0.3	0.0	66.0	7.5	19.2	1494.3	2.0
Unbounded	size	22	2	33	28	28	40	6
exchanges	weight	857	6	1546	1134	1275	1894	300
	longest c.	20	2	27	19	23	28	3
Chosen	#2-cycles	2	1	6	5	5	4	1
solution	#3-cycles	4	0	3	1	2	7	1
(NHSBT)	size	16	2	21	13	16	29	5
	weight	620	6	930	422	618	1168	288

- P. Biró, D.F. Manlove and R. Rizzi. Maximum weight cycle packing in directed graphs, with application to kidney exchange programs. Discrete Mathematics, Algorithms and Applications 1(4), pp:499-517, 2009.

2016-2021: COST Action on Kidney Exchanges

cost
EUROPEAN COOPERATION IN SCIENCE AND TECHNOLOGY

CA COST Action CA15210

European Network for Collaboration on Kidney Exchange Programmes

About one per thousand European citizens suffers from end stage renal disease. Living donor kidney transplantation is often the most effective treatment and the alternative of deceased donor kidney transplantation is severely limited by availability. As approximately 40% of living donors are incompatible with their specified recipient, several European countries have independently developed kidney exchange programmes (KEPs).

KEPs aim to match donors optimally to recipients for organ exchange within the population of recipient-donor pairs. Recent research shows that KEPs may greatly improve survival probabilities and quality of life, especially for recipients that are difficult to match. These recipients are disadvantaged disproportionally by the small scale of many national (or local) KEPs in Europe.

KEPs vary regarding the solutions provided for the problems in (i) the policy domain (prioritisation, equity, and accessibility); (ii) the clinical domain (clinical practice and evidence); and (iii) the optimisation domain (methods to solve the hard dynamic multi-criteria matching problems which take clinical evidence and health policy into account). Knowledge

COST Association COST Action CA15210

- Description
- Parties
- Management Committee

General Information*

Chair of the Action:
Prof Joris VAN DE KLUNDERT (NL)

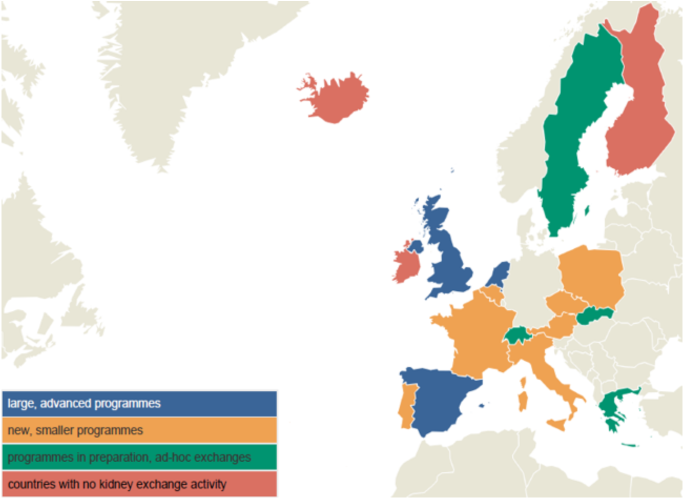
Vice Chair of the Action:
Dr David MANLOVE (UK)

Science officer of the Action:
Ms Estelle EMERIAU

Administrative officer of the Action:
Ms Carmencita MALIMBAN

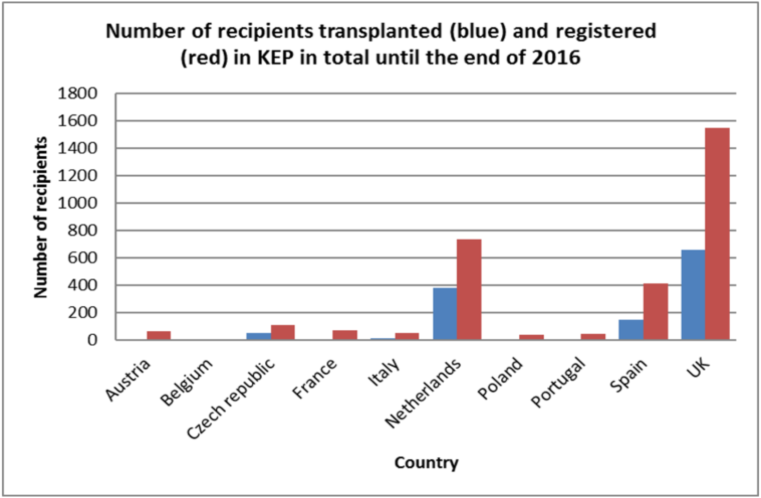
Downloads*

Kidney exchange programmes in Europe



- P. Biró, Bernadette Haase, and et al.: Building kidney exchange programmes in Europe – an overview of exchange practice and activities. Transplantation, 103 (7): 1514-1522, 2019.

Kidney exchange programmes in Europe



- P. Biró, Bernadette Haase, and et al.: Building kidney exchange programmes in Europe – an overview of exchange practice and activities. Transplantation, 103 (7): 1514-1522, 2019.

Details of the European KEPs

	Austria	Belgium	Czech Republic	France	Italy	Netherlands	Poland	Portugal	Spain	UK	Sweden	Switzerland
First exchange in KEP: 20XX	13	14	11	14	07	04	15	13	09	07		
Altruistic donor chains possible?	✓	✗	✓	✗	✓	✓	✗	✓	✓	✓	✗	✗
Compatible pairs/ couples participate?	✗	✗	✓	✗	✓	✓	✓	✗	✓	✓	✓	✓
Multiple donors register for one patient?	✓	✗	✓	✗	✓	✗	✓	✓	✓	✓	✓	✓
Incompatible transplants allowed within KEP?	✓	✗	✓	✗	✗	✗	✗	✗	✓	✓	✓	✓
Single lab carries out cross matching after virtual matching?	✓	✓	✓	✗	✗	✓	✗	✗	✗	✗	✗	✗
Simultaneous surgery required for an exchange in KEP?	✓	✓	✗	✓	✓	✓	✗	✓	✓	✓	✓	✓
Organs usually travel (O) or donors (D)?	D	O	-	O	O	D	O	O	O	O	O	D
Matching process every x months (NR=not regular)	NR	NR	3	3	NR	3	1	3	4	3	na	3
Longest exchange already conducted	3	3	7	2	2	4	3	3	3	3	na	na
Longest chain already conducted	na	na	6	na	6	2	na	na	6	3	na	na

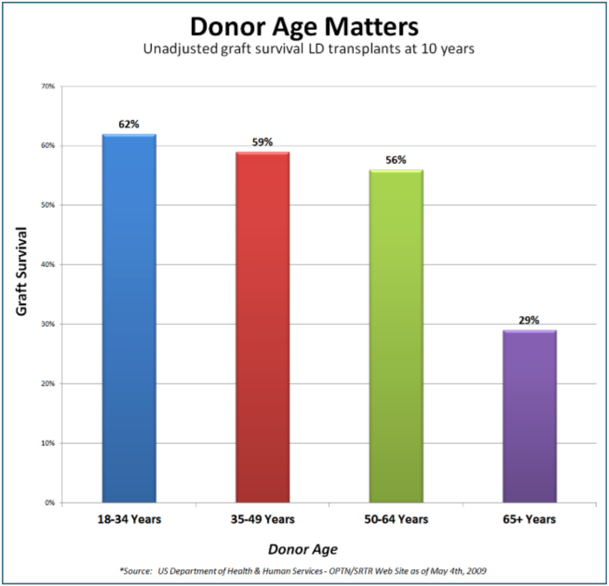
- P. Biró, Bernadette Haase, and et al.: Building kidney exchange programmes in Europe – an overview of exchange practice and activities. Transplantation, 103 (7): 1514-1522, 2019.

Optimisation in Europe: size vs quality

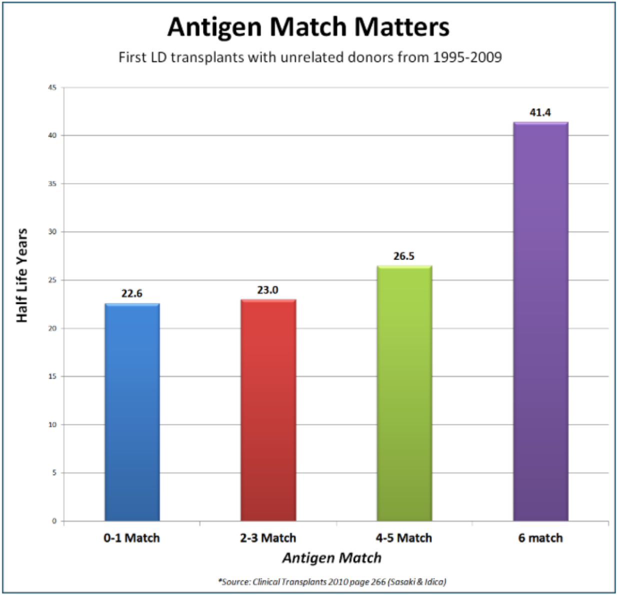
	Belgium	Czech Republic	Netherlands	Poland	Portugal	Spain	Sweden	United Kingdom
max size of solution	1	1	1	1	1	1	1	2
min lengths of the cycles	-	-	4	-	-	-	-	-
max # cycles selected	-	2	-	-	-	2	-	3
max # back-arcs	-	-	-	-	-	3	-	4
max # 2-cycles and 3-cycles with embedded 2-cycles	-	-	-	-	-	-	-	1
min# desensitisations	-	w	-	-	-	-	3	-
max HLA-matching	-	w	-	w	-	-	-	w
max DR-antigen matching in particular	-	w	-	-	-	-	-	-
min age-differences between the donors and patients	5	-	-	w	-	w	-	-
priority for paediatric patient	-	-	-	-	-	w	-	-
priority for patients not yet on dialyses	4	-	-	w	-	-	-	-
priority for highly sensitive patients	-	-	-	w	-	4	-	w
priority for O patients	-	-	-	w	-	-	-	-
priority for hard-to-match patients	3	-	3	w	w	w	2	-
priority for waiting time in KEP	-	-	6	-	-	w	-	w
priority for waiting time on the deceased WL	-	-	-	-	-	-	-	-
priority for time on dialyses	4	-	-	-	w	w	-	-
priority for same blood-group transplants	2	-	2	-	w	w	-	-
priority for O-to-O transplants	-	-	-	-	-	-	-	-
priority for pairs with AB-donors	-	-	-	-	-	w	-	-
max # of transplant centres in (long) cycles	-	-	5	-	-	-	-	-
priority for donor-patients in the same region	-	-	-	-	-	w	-	-
min the donor-donor age differences	-	-	-	w	w	-	-	w

- P. Biró, J. van de Klundert, D. Manlove, and et al.: Modelling and optimisation in European Kidney Exchange Programmes. European Journal of Operational Research, 291:447-456, 2021

NKR (US): quality incentives for compatible pairs



NKR (US): quality incentives for compatible pairs



Quality factors: acceptability thresholds in the UK

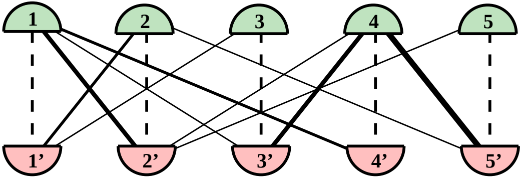
Significant differences in expected graft survival times based on:

- age of living donor
- HLA-matching between donor and patient
- whether patient needs desensitisation for ABOi transplants

UK practice: acceptability thresholds can be set for each of the above parameters by the individual patients/doctors!

Individual fairness vs social welfare

Patients needing transplant exchange their incompatible donors

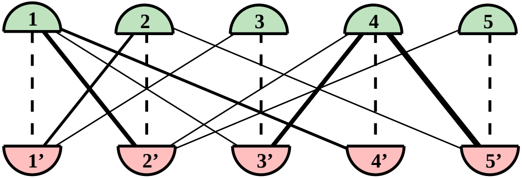


Qualities of transplants are determined by age and HLA-matching

$\sim$ expected lifetime gains $\sim$ graph survival times

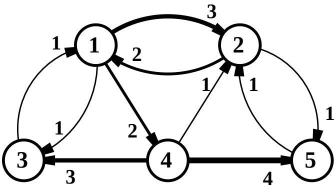
Individual fairness vs social welfare

Patients needing transplant exchange their incompatible donors



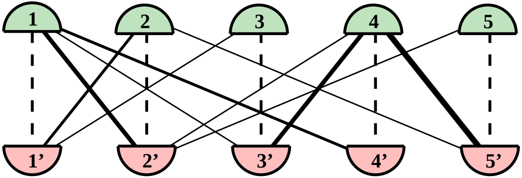
Qualities of transplants are determined by age and HLA-matching

$\sim$ expected lifetime gains $\sim$ graph survival times



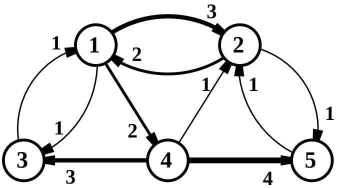
Individual fairness vs social welfare

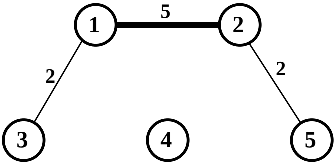
Patients needing transplant exchange their incompatible donors



Qualities of transplants are determined by age and HLA-matching

$\sim$ expected lifetime gains $\sim$ graph survival times

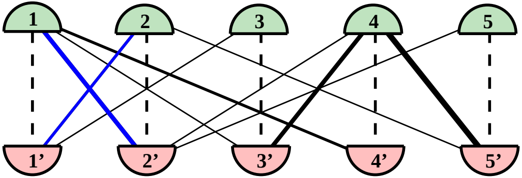




set of 2-way exchanges $\iff$ matching in an undirected graph

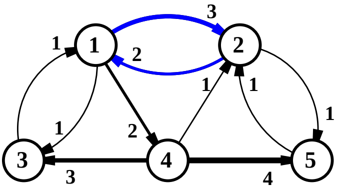
Individual fairness vs social welfare

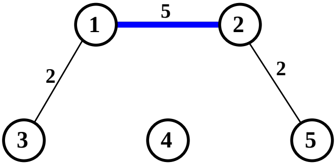
Patients needing transplant exchange their incompatible donors



Qualities of transplants are determined by age and HLA-matching

$\sim$ expected lifetime gains $\sim$ graph survival times



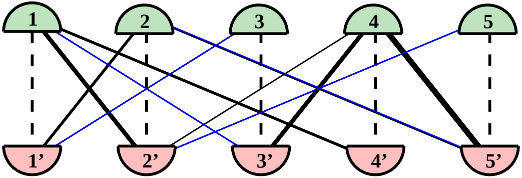


set of 2-way exchanges $\Longleftrightarrow$ matching in an undirected graph

quality transplants for some vs kidneys for more patients

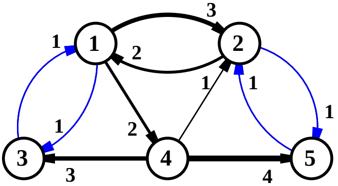
Individual fairness vs social welfare

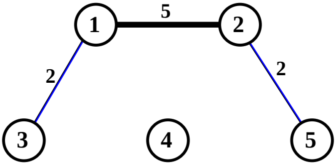
Patients needing transplant exchange their incompatible donors



Qualities of transplants are determined by age and HLA-matching

$\sim$ expected lifetime gains $\sim$ graph survival times



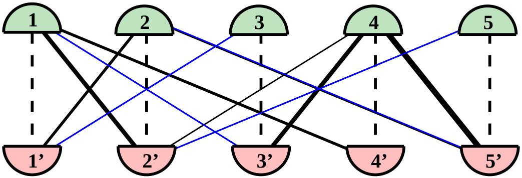


set of 2-way exchanges $\iff$ matching in an undirected graph

quality transplants for some vs kidneys for more patients

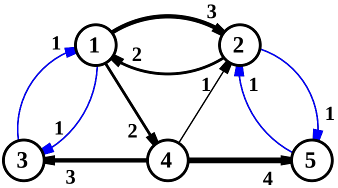
Individual fairness vs social welfare

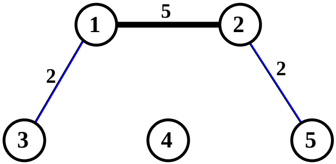
Patients needing transplant exchange their incompatible donors



Qualities of transplants are determined by age and HLA-matching

$\sim$ expected lifetime gains $\sim$ graph survival times





set of 2-way exchanges $\iff$ matching in an undirected graph

quality transplants for some vs kidneys for more patients

stable exchange (= core solution): no blocking cycle

Complexity of exchange problems

		exchanges		
		pairwise		
maximum size/weight	does exist?	**yes**		
	hard to find?			
stable	does exist?			
	hard to find?			

Complexity of exchange problems

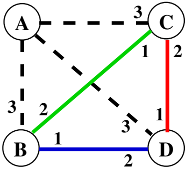
		exchanges		
		pairwise		
maximum size/weight	does exist?	yes		
	hard to find?	**P**		
stable	does exist?			
	hard to find?			

Edmonds (1967): Polynomial time algorithms for maximum size / maximum weight matching problem.

Complexity of exchange problems

		exchanges		
		pairwise		
maximum size/weight	does exist?	yes		
	hard to find?	P		
stable	does exist?	**may not**		
	hard to find?			

stable pairwise exchange = stable roommates



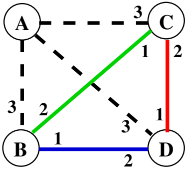
Gale and Shapley (1962):

Stable matching may not exist!

Complexity of exchange problems

		exchanges		
		pairwise		
maximum size/weight	does exist?	yes		
	hard to find?	P		
stable	does exist?	may not		
	hard to find?	**P**		

stable pairwise exchange = stable roommates

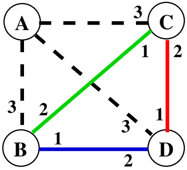


Gale and Shapley (1962):
Stable matching may not exist!
Irving (1985): A stable matching can be found in linear time, if one exists.

Complexity of exchange problems

		exchanges		
		pairwise		
maximum size/weight	does exist?	yes		
	hard to find?	P		
stable	does exist?	may not		
	hard to find?	P		

stable pairwise exchange = stable roommates



Gale and Shapley (1962):
Stable matching may not exist!
Irving (1985): A stable matching can be found in linear time, if one exists.

Abraham-Biró-Manlove (2006): The problem of minimising the number of blocking pairs is NP-hard.

Complexity of exchange problems

		exchanges		
		pairwise	**2-3-way**	
maximum size/weight	does exist?	yes	**yes**	
	hard to find?	P		
stable	does exist?	may not		
	hard to find?	P		

Complexity of exchange problems

		exchanges		
		pairwise	2-3-way	
maximum size/weight	does exist?	yes	yes	
	hard to find?	P	**NP-hard**	
stable	does exist?	may not		
	hard to find?	P		

Biró-Manlove-Rizzi (2009): Finding a maximum size/weight 2-3-way exchange is NP-hard, but there is a $O(2^{\frac{m}{2}})$-time exact algorithm. This was implemented for NHS Blood and Transplant in 2007 and used to compute optimal solutions subsequently.

- P. Biró, D.F. Manlove and Romeo Rizzi. Maximum weight cycle packing in directed graphs, with application to kidney exchange programs. Discrete Mathematics, Algorithms and Applications, 1 (4) : 499-517, 2009.
- P. Biró and E. McDermid. Three-sided stable matchings with cyclic preferences. Algorithmica, 58: 5-18, 2010. (COMSOC 2008)

Complexity of exchange problems

		exchanges		
		pairwise	2-3-way	
maximum size/weight	does exist?	yes	yes	
	hard to find?	P	NP-hard	
stable	does exist?	may not	**may not**	
	hard to find?	P	**NPc**	

Biró-Manlove-Rizzi (2009): Finding a maximum size/weight 2-3-way exchange is NP-hard, but there is a $O(2^{\frac{m}{2}})$-time exact algorithm. This was implemented for NHS Blood and Transplant in 2007 and used to compute optimal solutions subsequently.

Biró-McDermid (2010): Stable 2-3-way exchange may not exist, and the related problem is NP-complete, even for tripartite graphs.

- P. Biró, D.F. Manlove and Romeo Rizzi. Maximum weight cycle packing in directed graphs, with application to kidney exchange programs. Discrete Mathematics, Algorithms and Applications, 1 (4) : 499-517, 2009.
- P. Biró and E. McDermid. Three-sided stable matchings with cyclic preferences. Algorithmica, 58: 5-18, 2010. (COMSOC 2008)

Complexity of exchange problems: unbounded case

		exchanges		
		pairwise	2-3-way	**unbounded**
maximum size/weight	does exist?	yes	yes	**yes**
	hard to find?	P	NPc	
stable	does exist?	may not	may not	
	hard to find?	P	NPc	

Complexity of exchange problems: unbounded case

		exchanges		
		pairwise	2-3-way	unbounded
maximum size/weight	does exist?	yes	yes	yes
	hard to find?	P	NPc	**P**
stable	does exist?	may not	may not	
	hard to find?	P	NPc	

Graph Theory folklore: The problem of finding a maximum size/weight (unbounded) exchange is P-time solvable.

Complexity of exchange problems: unbounded case

		exchanges		
		pairwise	2-3-way	unbounded
maximum size/weight	does exist?	yes	yes	yes
	hard to find?	P	NPc	P
stable	does exist?	may not	may not	**yes**
	hard to find?	P	NPc	**P**

Graph Theory folklore: The problem of finding a maximum size/weight (unbounded) exchange is P-time solvable.

Shapley-Scarf (1972): Stable exchange always exists, one can be found by the Top Trading Cycle algorithm of Gale.
This was the original model in the seminal paper on Kidney exchange by Roth-Sönmez-Ünver (QJE 2005)!

New developments on stable exchanges

- Klimentova-Biró-Costa-Viana-Pedroso: Novel IP formulations for the stable kidney exchange problem (2021, arXiv)

-Computation of bounded length stable exchanges by IP techniques
-Measuring size vs stability tradeoffs

- Biró-Klijn-Klimentova-Viana: Shapley-Scarf Housing Markets: Respecting Improvement, Integer Programming, and Kidney Exchange (2021, arXiv)

-Respecting improvement property for stable exchanges: if a patient brings a better donor (e.g., younger or with a better blood type: $0 > A, B > AB$), or an additional donor, then in the TTC solution she must receive an exchange donor at least as good as before. We will discuss this paper in the next 15 mins!

Summary of our results

1. Proving the respecting improvement property for strong core and CE for unbounded exchanges, and examples for violations
2. New IP models for computing the strong core, CE, core
3. Simulations for measuring the price of fairness and the amount of respecting improvement violations for kidney exchange instances

Shapley-Scarf 1974 housing market model

A housing market (N, R) consists of
set of agents $N = \{1, \ldots, n\}$ with one house each, where
each agent $i \in N$ has complete and transitive (weak) preferences
R_i over the houses, where P_i denotes the strict relation.

An allocation x is a one-to-one re-assignment of the houses to agents, where x_i is the allotment of i.

A coalition $S \subseteq N$ blocks x if there is an allocation z s.t.
(1) $\{z_i : i \in S\} = S$ and
(2) for each $i \in S$, $z_i P_i x_i$.
x is in the core of the market if there is no blocking coalition.

Shapley-Scarf 1974 housing market model

A housing market (N, R) consists of
set of agents $N = \{1, \ldots, n\}$ with one house each, where
each agent $i \in N$ has complete and transitive (weak) preferences
R_i over the houses, where P_i denotes the strict relation.

An allocation x is a one-to-one re-assignment of the houses to agents, where x_i is the allotment of i.

A coalition $S \subseteq N$ weakly blocks x if there is an allocation z s.t.
(1) $\{z_i : i \in S\} = S$,
(2) for each $i \in S$, $z_i R_i x_i$, and
(3) for some $j \in S$, $z_j P_j x_j$.
x is in the strong core of the market if there is no weakly blocking coalition.

Shapley-Scarf 1974 housing market model

A housing market (N, R) consists of
set of agents $N = \{1, \ldots, n\}$ with one house each, where each agent $i \in N$ has complete and transitive (weak) preferences R_i over the houses, where P_i denotes the strict relation.

An allocation x is a one-to-one re-assignment of the houses to agents, where x_i is the allotment of i.

For price-vector p let p_i denote the price of object i. A competitive equilibrium is a pair (x, p) s.t.
(1) for each agent $i \in N$, object x_i is affordable, i.e., $p_{x_i} \leq p_i$ and
(2) for each agent $i \in N$, each object she prefers to x_i is not affordable, i.e., jP_ix_i implies $p_j > p_i$.
An allocation is a competitive allocation if it is part of some competitive equilibrium.

Shapley-Scarf 1974 housing market model

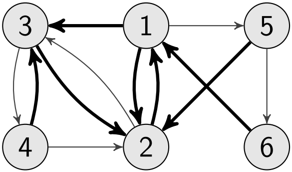
A housing market (N, R) consists of
set of agents $N = \{1, \ldots, n\}$ with one house each, where
each agent $i \in N$ has complete and transitive (weak) preferences
R_i over the houses, where P_i denotes the strict relation.

An allocation x is a one-to-one re-assignment of the houses to agents, where x_i is the allotment of i.

Wako (1999) showed that a competitive allocation can be characterised by the lack of antisymmetrically weakly blocking coalitions, that is a coalition $S \subseteq N$ s.t.
(1) $\{z_i : i \in S\} = S$,
(2) for each $i \in S$, either $z_i = x_i$ or $z_j P_j x_j$, and
(3) for some $j \in S$, $z_j P_j x_j$.
We also call the set of competitive allocation as Wako-core.

An example for core, CE, and strong core

Acceptability graph:



Preferences:

1	2	3	4	5	6
2,3	1	2	3	2	1
5	3	4	2	6	

Allocations:

$x^a = \{(1,3,2)\}$
$x^b = \{(1,2),(3,4)\}$
$x^c = \{(1,5,2),(3,4)\}$
$x^d = \{(1,3,4,2)\}$
$x^e = \{(1,5,6),(2,3,4)\}$

Solution sets:

strong core: $\{x^a\}$
CE/Wako-core: $\{x^a, x^b\}$
core: $\{x^a, x^b, x^c, x^d\}$
max size allocations: $\{x^e\}$

By definition strong core $\subseteq$ CE/Wako-core $\subseteq$ core

Shapley-Scarf (1974): housing market

Gale's Top Trading Cycles algorithm (TTC)

- Everybody points to the best house in the market (or one of the best houses if we have ties), we get at least one TTC
- Agents in a TTC exchange and then they leave the market
- We repeat the process in the remaining market...
  see an example at http://www.matchu.ai/

Shapley-Scarf (1974): housing market

Gale's Top Trading Cycles algorithm (TTC)

- Everybody points to the best house in the market (or one of the best houses if we have ties), we get at least one TTC
- Agents in a TTC exchange and then they leave the market
- We repeat the process in the remaining market...
  see an example at `http://www.matchu.ai/`
- The resulting solution is a competitive allocation.

  Proof: We set the prices of the houses decreasingly according to their removal order in the TTC...

  An allocation is competitive $\iff$ it can be obtained by TTC

Further theoretical results

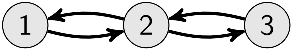
Roth-Postlewaite (1977): For strict preferences, the TTC algorithm returns the unique competitive allocation, which is also the unique strong core allocation.

Roth (1982): For strict preferences, the TTC algorithm is strategy-proof.

Ma (1994): For strict preferences, the TTC algorithm is the unique mechanism which is individually rational, Pareto-efficient and strategy-proof.

Quint-Wako (2004): For weak preferences, it is possible to find a strong core allocation efficiently, if there exists one.

An example for empty strong core:



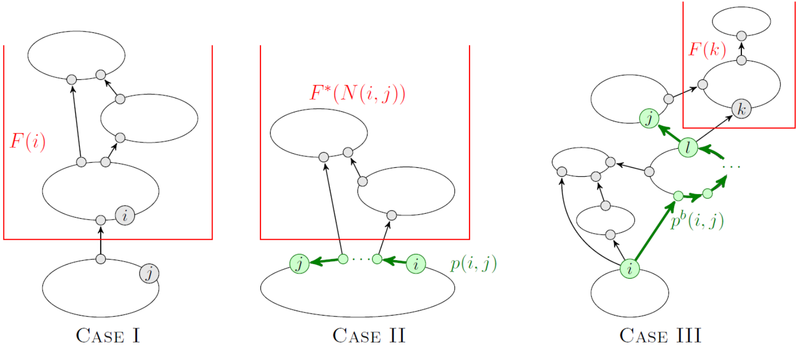
New results on the respecting improvement property

Biro-Klijn-Klimentova-Viana (2021): Suppose that the house of an agent i becomes better to another agent j, then

- for strict preferences, then the TTC solution in the new market can only be better for i than the TTC solution in the old market (RI-property)
- for weak preferences, if strong core solutions exist for both the old and new markets then the above respecting improvement property holds (conditional RI-property)
- for weak preferences, the best/worst competitive allocation in the new market is at least as good for i than the best/worst competitive allocation in the old market (RI-best/worst property)

Schlotter-Biro-Fleiner (2021): the RI-best property holds for core allocations even for partial orders (more general than weak preferences), but the RI-worst property is violated even for strict preferences

Sketch proof for respecting improvement property of TTC

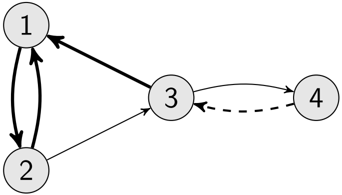


- Case I: agent *i* has left the market earlier - no effect
- Case II: they left the market at the same time - TTC cycle may get shorter, but *i* gets the same house
- Case III: *j* left the market earlier - *j* will be involved in a TTC earlier, she may get a better house

Violations of the respecting improvement property

Example for **max size unbounded exchanges**:

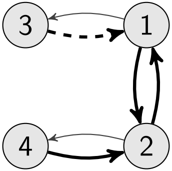
If house 3 becomes acceptable for agent 4, then 3 receives a worse house in the max size solution.



Violations of the respecting improvement property

Example for **max size pairwise exchanges**:

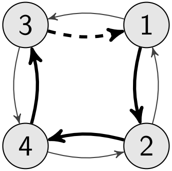
If donor 1 becomes acceptable for recipient 3, e.g., because recipient 1 brings a second donor, then she receives a worse kidney in the max size/weight solution.



Violations of the RI-worst property

Example for (strong) core pairwise exchanges, strict preferences:

If student 1 becomes acceptable for school 3, e.g., because she improves her score, then she receives a worse school seat in the school-optimal stable matching.



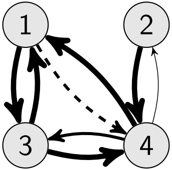
Balinski-Sönmez (1999): The student-optimal stable matching by the Gale-Shapley algorithm respects improvements for students.

Schlotter-Biro-Fleiner (2021): for strict preferences, the core/CE/strong core solutions satisfy the RI-best property (generalisation for the roommates problem)

Violations of the RI-best property

Example for (strong) core pairwise exchanges, weak preferences:

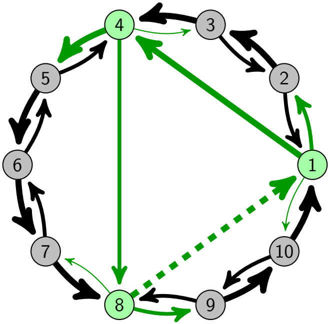
If agent 1 becomes acceptable for possible roommate 4, then she receives a strictly worse roommate in the unique stable matching.



Violations of the RI-best property

Example for (strong) core 3-way exchanges, strict preferences:

If house 1 becomes acceptable for agent 8, then 1 receives a strictly worse allotment in the best core allocation than before.



IP formulations with edge variables

$$y_{ij} = \begin{cases} 1 & \text{if agent } i \text{ receives object } j; \\ 0 & \text{otherwise.} \end{cases}$$

$$\sum_{j:(i,j)\in E} y_{ij} = 1 \qquad \forall i \in N \qquad (1)$$

$$\sum_{j:(j,i)\in E} y_{ji} = 1 \qquad \forall i \in N \qquad (2)$$

$$y_{ij} \in \{0,1\} \qquad \forall (i,j) \in E \qquad (3)$$

IP: no blocking constraints for core/Wako/strong core

Quint-Wako (2004): IP formulations for all permutations re-written for cycles (works for both bounded&unbounded)

core:

$$\sum_{(i,j)\in A(c)} \sum_{k:kR_ij} y_{ik} \geq 1 \qquad \forall c \in \mathcal{C} \tag{4}$$

CE/Wako-core:

$$\sum_{(i,j)\in A(c)} y_{ij} + |c| \cdot \left[\sum_{(i,j)\in A(c)} \sum_{k:kR_ij,k\neq j} y_{ik} \right] \geq |c| \qquad \forall c \in \mathcal{C} \tag{5}$$

strong core:

$$\sum_{(i,j)\in A(c)} \sum_{k:kI_ij} y_{ik} + |c| \cdot \left[\sum_{(i,j)\in A(c)} \sum_{k:kP_ij} y_{ik} \right] \geq |c| \qquad \forall c \in \mathcal{C} \tag{6}$$

IP: new compact formulations for unbounded case

We introduce prices (corresponding to a topological order):

$$p_i \in \{1, \ldots, n\} \qquad \forall i \in N \tag{7}$$

core:

$$p_i + 1 \leq p_j + n \cdot \sum_{k:kR_ij} y_{ik} \qquad \forall (i,j) \in E \tag{8}$$

CE/Wako-core: in addition to the above constraints

$$p_i \leq p_j + n \cdot (1 - y_{ij}) \qquad \forall (i,j) \in E \tag{9}$$

strong core: in addition to the above constraints

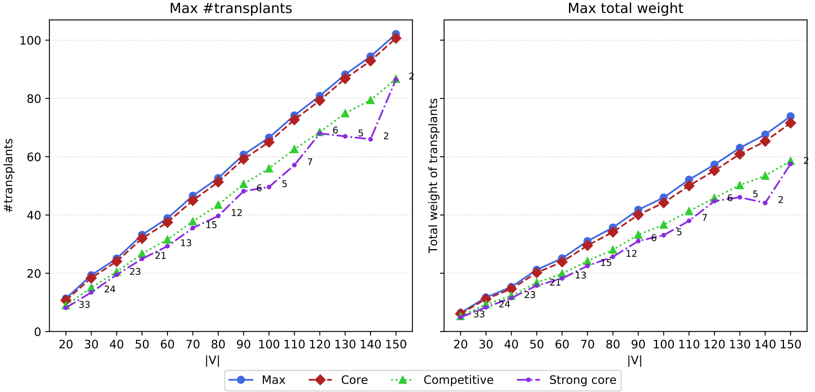
$$p_i \leq p_j + n \cdot \left(\sum_{k:kP_ij} y_{ik} \right) \qquad \forall (i,j) \in E \tag{10}$$

Computer simulations for bounded/unbounded cases

Testing strong core/Wako-core/core/**max size/max weight** solutions on realistic kidney exchange instances

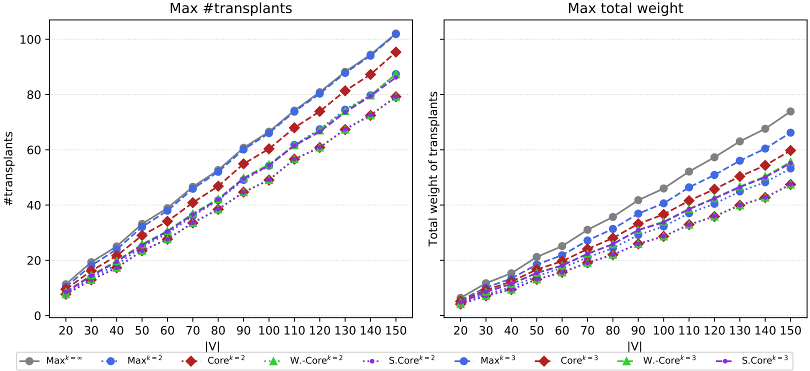
- efficiency of IP formulations
- price of fairness (i.e., size vs stability)
- counting the number of violations of the RI-best property for difference solution concepts

Optimality vs stability tradeoff (i.e., price of fairness)



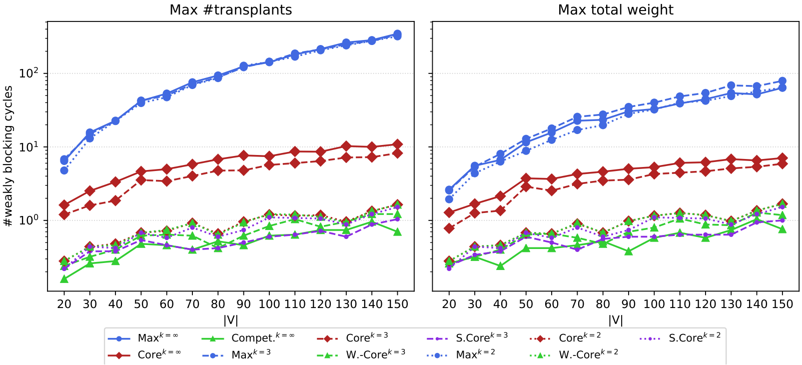
Number of transplants (left) and total weight of transplants (right) for unbounded length exchanges and weak preferences.

Optimality vs stability tradeoff (i.e., price of fairness)



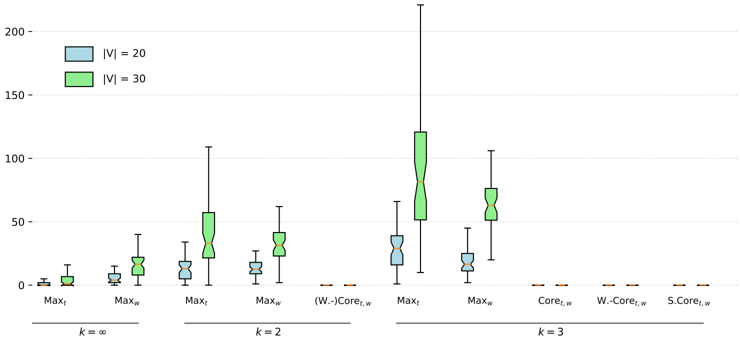
Comparison of the number of transplants (left) and the total weight of transplants (right) for bounded length exchange cycles ($k = 2, 3$) and weak preferences.

Optimality vs stability tradeoff (i.e., price of fairness)



Number of weakly blocking cycles of size $l = 2$ for solutions with maximum number of transplants (left) and maximum total weight of transplants (right), for unbounded exchange cycles and exchange cycles of size up to $k = 2$ and $k = 3$ for weak preferences.

Violations of the RI-best property

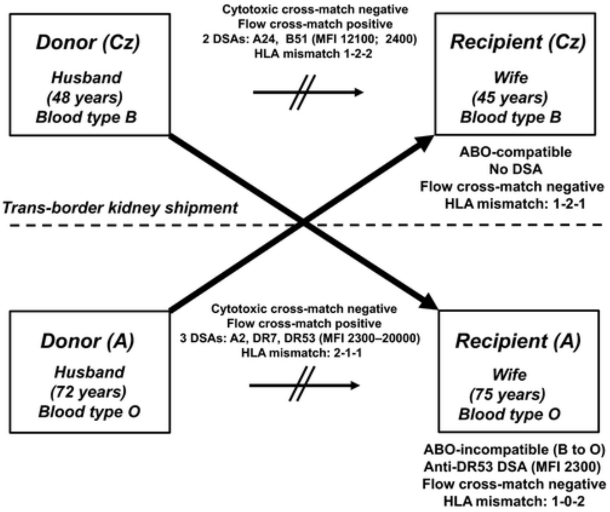


Number of violations of the RI-best property for instances of sizes $20, 30$, weak preferences.

Fairness concepts

1. fairness for individual patients vs social welfare
2. **fairness for countries in international KEPs**

International exchanges: Austria-Czech Rep. 2016



- G.A. Böhmig, J. Fronek, A. Slavcev, G.F. Fischer, G. Berlakovich, and O. Viklicky. Czech–Austrian kidney paired donation: First European cross-border living donor kidney exchange. Transplant International, 30(6):638-639, 2017.

Tommy Andersson

Május 22., 19:34 ·

I am very happy to report that it is now official that the first match run for the Scandinavian kidney exchange programme STEP was conducted on April 9 (before January 1, 2019, only Sweden was part of the programme). Scandiatransplant writes that "several cycles have been identified" (I know how many cycles that was identified, but this is not public information). This is great news for all patients in Sweden, Denmark and Norway in need of a kidney transplant. It is also a great achievment of the hard working transplant community in these countries (transplant surgeons, tissue typers, kidney coordinators, and all other persons involved). In the long run, this will save many many lives ❤️🇩🇰🇳🇴🇸🇪

Read more at:

SCANDIATRANSPLANT.ORG

www.scandiatransplant.org

👍❤️😮 124 15 hozzászólás 2 megosztás

International kidney exchanges in Europe

Deceased organ sharing schemes:

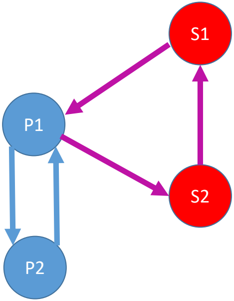
- Eurotransplant (Austria, Belgium, Croatia, Germany, Hungary, Luxembourg, Netherlands, Slovenia)
- Scandiatransplant (Denmark, Estonia, Finland, Iceland, Norway, Sweden)
- South Transplant Alliance (France, Italy, Spain)

Living kidney exchanges:

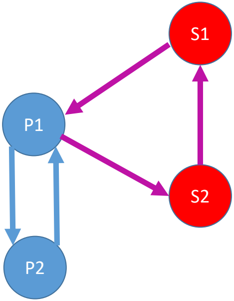
- Austria-Czech Rep. since 2016 (+Czech-Israel since 2019)
- Italy-Spain-Portugal since 2018
- Scandiatransplant (STEP): Sweden-Denmark since 2019 (Finland and Norway joining in 2021)

The Italy-Spain-Portugal collaboration has **sequential match runs**: national runs first, and then the international run for the remaining pairs (conservative approach)

Fairness at country level, an example



Fairness at country level, an example

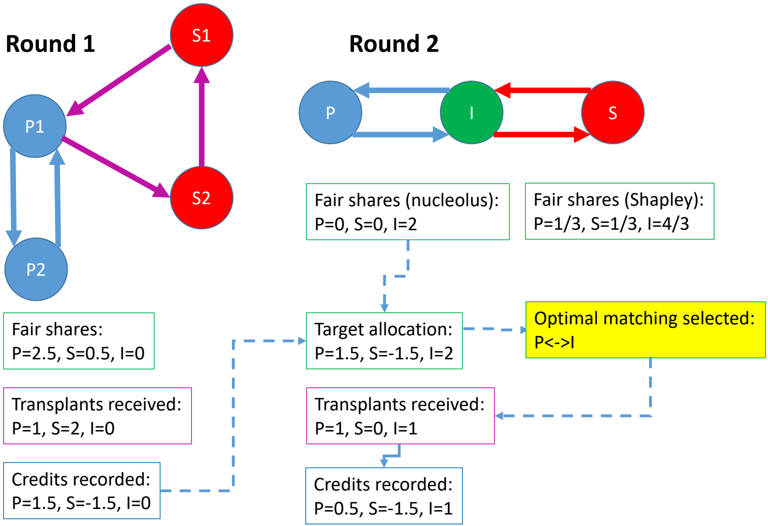


Fair shares:
P=2.5, S=0.5, I=0

Transplants received:
P=1, S=2, I=0

Credits recorded:
P=1.5, S=-1.5, I=0

Fairness at country level, an example



Literature review

Multi-country compensation schemes (European focus)

- X. Klimentova, N. Santos, J.P. Pedroso, and A. Viana. Fairness models for multiagent kidney exchange programs. Omega, 2020
- Biró-Kern-Paulusma-Pálvölgyi: Generalized Matching Games for International Kidney Exchange, AAMAS-2019
- Gyetvai-Biró-Klimentova-Pettersson-Viana: Compensation scheme with Shapley value for multi-country kidney exchange programmes ECMS-2020

Multi-hospital kidney exchanges (US focus)

- I. Ashlagi and A.E. Roth. New challenges in multihospital kidney exchange. American Economic Review, 102(3):354–359, 2012.
- I. Ashlagi and A.E. Roth. Free riding and participation in large scale, multihospital kidney exchange. Theoretical Economics, 9(3), 2014.
- I. Ashlagi, F. Fischer, I.A. Kash, and A.D. Procaccia. Mix and match: A strategyproof mechanism for multi-hospital kidney exchange. Games and Economic Behavior, 91:284–296, 2015.
- P. Toulis and D.C. Parkes. Design and analysis of multi-hospital kidney exchange mechanisms using random graphs. Games and Economic Behavior, 91:360-382, 2015.
- C. Hajaj, J. Dickerson, A. Hassidim, T. Sandholm, and D. Sarne. Strategy-proof and efficient kidney exchange using a credit mechanism. In Proceedings of AAAI-2015, 921–928, 2015.
- N. Agarwal, I. Ashlagi, E. Azevedo, C.R. Featherstone, and O. Karaduman. Market failure in kidney exchange. American Economic Review, 2019.

Compensation system plans: Europe vs US

European multi-country model, proposed by Klimentova et al.:

1. reporting full pools is an assumption by protocol
2. quarterly runs: select the matching (set of exchanges/chains) according to target allocations
3. pool compositions may differ across countries significantly
4. compensation scheme based on country-valuations

US (NKR) multi-hospital model, see Agarwal et al. (AER, 2019):

1. hospitals need incentive to report full pools
2. online matching: select the new exchange cycle/chain according to the credits
3. similar pool compositions are expected in the hospitals
4. credit system based on pair valuations (easy/hard-to match)

New papers on international kidney exchanges

- X. Klimentova, N. Santos, J.P. Pedroso, and A. Viana. Fairness models for multiagent kidney exchange programs. Omega 2020

Simulations for international KEPs with compensation schemes.

- Gyetvai-Biró-Klimentova-Pettersson-Viana: Compensation scheme with Shapley value for multi-country kidney exchange programmes, ECMS-2020

Follow-up for the Shapley value as target solution.

- Biró-Kern-Paulusma-Pálvölgyi: Generalized Matching Games for International Kidney Exchange. AAMAS-2019

Theoretical paper on pairwise exchanges only, studying the complexity of finding a core solution and target matchings.

- Benedek-Biró-Kern-Paulusma: Computing Balanced Solutions for Large International Kidney Exchange Schemes

Follow-up simulations for nucleolus, Shapley-value, and other concepts for large number of countries.

New European project: KEP-SOFT

COST Innovators Grant (1 Nov 2021 - 31 Oct 2022)

KEP-SOFT will deliver a new software tool, building on the ENCKEP prototype and drawing on the existing expertise that has been built up within ENCKEP, which includes clinicians, policy makers, optimisation experts, computer scientists, mathematicians and economists.

KEP-SOFT will be made freely available as a resource to the European transplantation community, and to National Transplantation Organisations.